HEDGING NET FIRM INCOME FOR FLORIDA DAIRY PRODUCERS: A CASE STUDY WITH AN APPLICATION OF THE CONDITIONAL VALUE AT RISK

By

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To my parents.

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Production risk and coverage under MILC impact the producer's ability to hedge. The MILC program is a direct-payment program designed to help producers cope with price risk. Larger producers quickly exhaust coverage under MILC. Furthermore, production risk varies by season for dairy producers. Various degrees of production risk and MILC coverage ensure that producers face a dynamic risk environment. This research attempts to estimate the producer's minimum risk hedge ratio using class III futures and options under various MILC, production risk and capital structure scenarios.

Monte Carlo simulation was used to forecast the difference between milk revenue and expenses. The mailbox price was estimated as the sum of the blend price and over order premium. The stochastic process used to define the blend price was based on a modified binomial tree that modeled price movements in the class III market. The binomial tree was modified in order to compensate for the idiosyncrasies of the dairy industry, namely the milk price-support program. Class I, II, and IV prices were modeled

relative to the class III prices using historical data. The over order premium was estimated using historical data. Stochastic utilization and milk production per cow estimates were calculated using historical data. Producer expenses were defined using survey data. Linear regression was used to determine the sensitivity of total cost to changing levels of production.

Our study compares the minimum risk level attainable using futures and options under various policy, production risk and capital structure scenarios. This research has found that the minimum risk hedge ratio decreases drastically when the producer is completely covered under MILC. The preceding result can be explained by the fact that that the deficiency payments received under MILC are similar to the payments received from a European put option. This yields a substitution effect limiting the effectiveness of class III futures and options. Production risk also decreases the minimum risk hedge ratio although not nearly as drastically. The firm's use of debt shifts the risk measure by the amount the producer pays in interest. The producer's ability to risk balance is limited by the risk surface faced.

CHAPTER 1
COMPLEXITIES OF RISK MITIGATION FOR FLORIDA PRODUCERS

Dairy Product Utilization

The distinct utilization of milk in Florida coupled with the complex milk pricing scheme and dynamic policy environment complicate hedging for Florida dairy producers. Consumption of dairy products is classified into one of four classes. According to the Federal Milk Marketing Order Statistics (FMMO) 2002 Summary (USDA-AMS 2002), 88.93% of Florida's milk production was used to satisfy class I demand. Florida's dependence on the class I market distinguishes it from the rest of the nation, which contributes an average of only 36.67% of production to class I utilization (USDA-AMS 2002). Florida is further distinguished by its contribution to the class III market. Only 2.02% of Florida's production was used to satisfy class III milk demand, whereas on the national level the class III market consumed 44.38% of domestically produced milk representing the single largest utilization for US milk (USDA-AMS 2002).

Milk Pricing

Milk prices are a function of three factors. The Federal Price-support policy (price floor), the Federal Milk Marketing Order (FMMO) policy, and the over order premium.

The price-support policy is implemented through government purchases of storable products. The Commodity Credit Corporation (CCC) purchases butter, nonfat dry milk (NFDM), block cheddar, and barrel cheddar. Theoretically the CCC buys products during times of low prices and sells back product during periods of high prices (Bailey 1997) thus acting as a market stabilizer benefiting both consumers and producers. The

prices paid by the CCC are parameters calculated in order to ensure a price of $9.90 per hundredweight of delivered raw milk standardized at 3.67% butterfat. The relationship between the support price for butter and nonfat dry milk is called the butter powder tilt. If the CCC accumulates product and does not foresee prices elevating to a level where the product can be sold, the USDA can opt to adjust the relationship between the price of NFDM and butter.

The FMMO sets minimum prices based on utilization and geography. The milk price a processor can purchase raw milk at is based on the processor's utilization of the raw product. Processors that bottle milk pay a higher price and processors that make manufactured products, such as cheese and butter, pay a lower price. The FMMO system works in a similar manner as the milk price-support program (both establish a minimum price), except for the fact that the FMMO is tied to wholesale market prices of dairy products and adjusts monthly to compensate for changes in market conditions. The marketing order establishes a blend price which is based on utilization in each of the four classes and is equal to the minimum price a producer can be paid for delivered raw milk. The difference between the milk price-support and the blend price is that the blend price is adjusted monthly to reflect changes in monthly wholesale product demand while the milk price-support can only be changed through a congressional mandate.

The difference between the blend price for a particular order and the actual price received by producers is called the over order premium. The over order premium fluctuates with supply and demand conditions and results from bargaining between farmers and fluid milk processors (Prasertsri 2002).

Impact of Policy Shifts

The dairy industry is changing and will continue to change as government continues to adopt policies which vary direct and indirect agricultural price-supports. Legislators have argued that current dairy policy encourages over production, costs taxpayers millions and encourages production by inefficient farms. There are also many good reasons for keeping the price-support system.

The price-support system promotes price stability for producers and ensures that quality dairy products will be produced and readily available to consumers. Despite the positive effects of the price-support system, producers need to be able to adjust to volatile prices if the government policy changes. Milk prices became increasingly volatile after the passage of the 1996 Federal Agriculture Improvement Reform Act (FAIR Act).[1] The FAIR Act committed the government to a decreased role in the agricultural arena by systematically decreasing the support price from $10.35 in 1996 to $9.90 in 1999. Despite the increased protection offered by the 2002 farm bill, producers need to be able to adjust if government policy changes.

Recent Policy Changes

Recently, Congress introduced the Milk Income Loss Contract Program (MILC) in the 2002 farm bill. The MILC program compensates producers 45% of the difference between the Boston class I price and $16.94 per hundredweight (Westcott et al. 2002). Compensation is limited to 2.4 million pounds per dairy per year. This translates into full compensation for a 132 cow dairy where cows average 50 pounds of milk per day. The

[1] For a more complete discussion on emerging price volatility see: Blayney, D. P., A. C. Manchester, J.J. Miller, R. P. Stillman and D. A. Torgerson. "Price Volatility In the Dairy Industry and Its Causes." *Paper presented at Price Instability and Risk Management in the Dairy Industry Conference* (1998).

average dairy farm in the Southeast is much larger than the national average (Blayney 2002). This means that many of Florida's larger dairy producers will exhaust coverage under MILC prior to year's end.

The MILC program replaced the Dairy Options Pilot Program (DOPP). The DOPP was authorized under the Federal Agriculture Improvement and Reform Act of 1996 and was suppose to serve as a tool to help producers make the transition to a marketplace that was free from government interference. The program was designed to help producers learn how to use options to set their own price floor. The producer was able to purchase a subsidized put option, which ensured a minimum price for their class III milk. The put option gives producers the right to sell the underlying futures contract at the given strike price before the expiration date.

The profitability of a dairy farm is determined by the producers net firm income which equals revenues minus expenses. However, most congressionally mandated risk management programs, such as DOPP and MILC, address only the revenue portion of net income, discounting entirely the impact of expenses. In this study it is assumed that the distribution of net income defines risk for the dairy producer. This methodology emphasizes a holistic approach to risk management by considering the relationship between volatile revenue and expense streams. Dairy producers are able to mold the distribution of net income using class III futures and options.

Short- versus Long- Run Implications

Producers can mitigate changes in price risk in one of two ways. In the long run producers can reduce debt levels to reduce interest loads, and therefore reduce financial risk. The reduced financial risk will offset the increased business risk (resulting from increased price volatility), and the producer will still be faced with the same total risk

levels. In the short run, producers will still need to react, but it is unlikely that they will be able to reduce debt levels immediately, because of the inability to access the equity markets. The existing corporate finance theory on capital structure assumes frictionless capital markets, which allows the firm to have a flexible capital structure (Copeland and Weston 1988). For large firms, this may be an appropriate assumption but for small firms (such as dairies), this assumption is not appropriate. Therefore, in the short run, producers will have to deal directly with financial risk. If business risk increases as a result of changing government policy, then producers may keep existing debt levels and mitigate the business risk by going directly to the futures and options markets. Futures and options contracts can be used to reduce business risk and thus balance total risk.

Problem Statement

The Florida milk marketing order is highly dependent on the class I market. The Chicago Mercantile Exchange offers futures and options contracts based on the class III market. The problem facing Florida's producers is whether or not it is feasible to cross hedge net income based on the class I market with a portfolio of futures and/or options contracts that are based on the class III formula price. Our study considers the minimum risk level attainable to the Florida producer under different scenarios.

Research Objectives

Our study will determine if it is economically feasible to implement a more efficient marketing plan for Florida's dairy producers and show how the optimal marketing plan is a function of the dairy's capital structure. In order to accomplish this the following specific objectives must be addressed:

- Perform a literature review of risk measures, risk mitigating tools, and strategies used by firms
- Determine the historical variability of class utilization by quarter
- Establish the basis relationship between the futures and the spot market for Florida producers
- Develop a hedging strategy using class III milk futures and options
- Model the stochastic mailbox price
- Investigate production risk by month for Florida dairy producers
- Demonstrate the relationship between milk production and expenses
- Compare the historical class III implied volatility with the current and future implied class III volatilities
- Value the protection offered to Florida producers by the milk-price floor
- Evaluate the effectiveness of hedging net firm income for Florida producers not enrolled in the Milk Income Loss Contract (MILC) program under production certainty and production risk
- Evaluate the effectiveness of hedging net firm income prices for Florida producers enrolled in the MILC program under production certainty and under production risk
- Evaluate the effectiveness of using hedging to balance risk under varying capital structures for producers enrolled in MILC under production certainty and production risk
- Evaluate the effectiveness of using hedging to balance risk under varying capital structures for producers not enrolled in MILC under production certainty and production risk

CHAPTER 2
LITERATURE REVIEW

According to Harwood et al. (1999, p. 2), "Risk is the possibility of adversity or loss and refers to 'uncertainty that matters.' Consequently, risk management involves choosing among alternatives to reduce the effects of risk. It typically requires evaluating tradeoffs among changes in risk, expected returns, entrepreneurial freedom, and other variables. Understanding risk is a starting point to help producers make good management choices in situations where adversity and loss are possibilities."

This chapter will be divided into five sections. The first section of this chapter will discuss various types of business risk. The second section will define financial risk. The third section will consider the relationship between business and financial risk from a risk balancing perspective. The fourth sections will provide an overview and discuss the empirical applications of the variance, value at risk and conditional value at risk measures of risk. Finally the fifth section will discuss the contract specification on milk futures and options.

Business Risk

Production Risk

Harwood et al. (1999) defined several different types of business risk. Production risk is a function of natural factors such as weather, soil type, and region. Producers can protect themselves from production risk by using different types of technologies. Insecticides and herbicides can be used to protect against losses from insects and weeds, whereas irrigation and drainage can limit producers' dependence on weather. The

importance of irrigation on reducing yield risk can be clearly seen when the categorical risk rankings are contrasted for producers in regions where irrigation is widely available (such as California), with regions where it is not (such as the Midwest) (Harwood et al. 1999). Blank (1995) found yield risk was less important than price risk for California producers, a result that differs from surveys of Midwestern producers (who rank yield risk as a larger concern than price risk). Yield insurance is also available and may be used by farmers to protect against losses in regions where production is uncertain. In some regions, yield risk can be offset by price risk. For example, in some areas of the corn belt, when yields drop as a result of weather, the yields for everybody else in the region are generally adversely affected as well. Because production in this region represents a large proportion of the domestic corn supply, prices have a tendency to go up, protecting the farmer from serious drops in revenue.

Quantifying yield risk can be accomplished by determining the production trend for a particular farm or locality. The trend is described as known variation (or systematic variation that results from adopting of superior technologies, such as improved hybrids and more effective fertilizers). Harwood (1999) defines deviations from the trend as the source of yield risk. Yield risk can be found by forming the frequency distributions of corn yields from the trend. The frequency distribution allows the farmer to determine the probability that production will be below some threshold amount. This is important, particularly for the farmer who seeks to use futures and options to manage basis risk. An understanding of yield risk can minimize the probability of the disastrous consequences of hedging more than what is produced.

Price Risk

The farmer also faces price risk both from the inputs used and the outputs produced. The farmer can try to limit exposure to price risk by using forward pricing. The farmer has several different instruments at his disposal to deal with price risk. The forward contract is perhaps the most favored tool used to lock in a future price because of its flexibility and lack of basis risk. The farmer can enter into a forward contract (with a grain elevator, milk cooperative, etc.) and lock in a price for whatever quantity desired. Grain farmers can lock in a profit margin by merely forward pricing their output price, because the price of their inputs (such as fertilizer and seed) do not vary much. Livestock farmers and dairy producers should focus on locking in both their output and input price because of their reliance on commodities as inputs.

The farmer can also use a futures contract to lock in a price, or use an options contract to lock in a price floor. Use of either futures or options is limited by contract specifications, and subjects the farmer to some sort of basis risk. Basis risk is the variation of the difference between the futures (when the futures contract is offset) and the cash price (Hull 2000). The farmer must determine whether to accept the basis risk (associated with hedging) or the price risk (of selling on the cash market).

Daily price volatility is measured by finding the natural log of successive prices and multiplying the standard deviation of the volatility estimates by the square root of the number of observations in the month (Harwood et al. 1999).

Other Sources of Risk

The farmer is also exposed to an assortment of other risks that fall under the business-risk category. There is always institutional risk, or the risk that new laws will be passed that are unfriendly to producers. Different administrations weight the value of

price-supports, subsidies, and environmental compliance differently. As a result, farmers are constantly adapting to changing government regulations and laws.

Farmers are also exposed to litigation risk. Farming is a relatively dangerous occupation, and the risk of lawsuits from injuries can place a substantial financial burden on farmers. Some types of farming require significant capital outlay exposing the producer to the risk of technological innovation that makes the current technology obsolete. Most farms in the United States are run by families, and are therefore exposed to the risk of changing family relationships caused by divorce, sickness or death. Any of the factors can seriously strain the farm manager's ability to cope with day-to-day responsibilities he has on the farm.

Financial Risk

Collins (1985) defines the business risk as the variance of the return on equity and the financial risk as the proportional increase in the variance of return on equity that results from the producer's choice of capital structure. The variance is just one type of risk measure that will be discussed in this chapter. Additional types of risk measures include the value at risk and conditional value at risk for which the producer's financial risk can be similarly defined. In addition, the return on equity is just one type of profitability measure that is used, alternatives to the return on equity might include the producer net firm income or net income.

Farmers' Degree of Concern about Different Types of Risk

The ARMS data were developed from a nationwide survey of farmers. Farmers were asked to report their level of concern about various risk categories including price, yield, or livestock production risk, ability to adopt new technology, lawsuits, changing consumer preferences and changes in government laws and regulations. Dairy producers

ranked price risk as their greatest concern, followed by production risk. Beef, cotton, soybean, and corn producers also reported high levels of concern about yield and price risk relative to the other categories (Harwood et al. 1999). Findings were consistent with those of Jose and Valluru (1997) (that yield and price risk concerned Midwestern and Great Plains farmers the most). Results of the ARMS survey were also paralleled by a 1991 survey of Purdue Top Farmer Workshop participants. Patrick et al. (1998) recognized yield and price risk as the most important concern for 1991 workshop participants.

The ARMS survey results found that dairy, cotton, soybean, and corn producers ranked institution risk as the third most important form of risk they face; while beef producers ranked institutional risk as more important than yield risk, but less important than price risk (Harwood et al. 1999). This difference was attributed to growing industry concerns about environmental regulations. Dairy, beef, cotton, soybean, and corn producers all seemed relatively less concerned about their ability to adopt technology, about litigation risk, and about changing consumer preferences.

Risk Balancing

The proceeding arguments for more efficient marketing strategies are based on the premise that a tradeoff exists between financial and business risk. The risk balancing formulation presented by Gabriel and Baker (1980) is based on the notion that producers exhibit lexicographic preferences, maximizing return subject to a predetermined risk level. Total risk equals financial risk plus business risk. If the government initiates a price-support program, then the level of business risk decreases. The level of total risk therefore also decreases. Producers will assume a more leveraged position in order to increase the level of financial risk and thus reduce the slack in the risk constraint. The

following section develops risk measures initially proposed by Gabriel and Baker (1980). The second part presents a review of literature that has tested the risk balancing hypothesis empirically.

Business risk can be measured for a particular industry or firm, by considering the probability distribution of cash flow before servicing debt. According to Gabriel and Baker (1980) business risk for a particular firm can be calculated as

$$BR = \frac{\sigma_1}{\bar{cx}}, \tag{2-1}$$

where σ_1 equals the standard deviation of cash flows before servicing debt and the expected cash flows before servicing debt equals $\bar{cx}$.

The second category of risk, financial risk, is a function of the fixed payments required to service debt and business risk. As a firm increases the level of borrowing, its leveraged position increases, raising the firm's breakeven level of production. The firm's level of financial risk

$$FR = \frac{\sigma_2}{\bar{cx} - I} - \frac{\sigma_1}{\bar{cx}}, \tag{2-2}$$

where the standard deviation of cash flows from the levered firm before debt is serviced equals σ_2 and I equals the fixed payments required to service debt(Gabriel and Baker 1980).

The relationship between business risk and financial risk is addressed in the risk-balancing literature. Finding a common denominator and then factoring the expected cash flows before servicing debt, an alternative expression for financial risk can be derived that enables the evaluation of three different scenarios

$$FR = \frac{\sigma_2 - \sigma_1}{\overline{cx} - I} + \frac{\sigma_1}{\overline{cx}}\left(\frac{I}{\overline{cx} - I}\right). \tag{2-3}$$

The first scenario results when the standard deviation of cash flows before servicing debt for the unlevered firm is equal to the standard deviation of cash flows before servicing debt for the levered firm. This occurs when $\sigma_1 = \sigma_2$ (Gabriel and Baker 1980) and the expression for financial risk collapses to

$$FR = \frac{\sigma_1}{\overline{cx}}\left(\frac{I}{\overline{cx} - I}\right). \tag{2-4}$$

However, if the manager adopts a different, less risky marketing strategy after assuming the leveraged position (such that the standard deviation of cash flows before servicing debt is not equal for the levered and unlevered firm), then $\sigma_1 > \sigma_2$ (Gabriel and Baker 1980), and the level of financial risk is lower than it would have been had the producer not made the change (Gabriel and Baker 1980).

The final scenario results when the producer's level of financial risk is amplified by the decision to assume a levered position. When this scenario occurs, the producer adopts a more risky strategy, such that $\sigma_1 < \sigma_2$ (Gabriel and Baker 1980). This scenario results when the standard deviation of cash flows before servicing debt is greater for the levered firm than it is for the unlevered firm.

When the first scenario is assumed (such that the variation in cash flows is independent of financing) then business plus financial risk yields total risk expression for total risk collapses to

$$TR = \frac{\sigma_1}{\overline{cx}} \frac{\overline{cx}}{cx - I} \quad . \tag{2-5}$$

Gabriel and Baker (1980) were able to demonstrate (using linear regression) that changes to $\left(\frac{I}{\bar{c}x - I}\right)$,result when government policies affect the level of business risk. This is the component of equation (2-4) that balances financial risk with business risk. This suggests that producers are using financial risk to balance changes in business risk.

Using USDA Farm Income, Balance Sheet and Farm Real Estate Market Development Data, Gabriel and Baker (1980) found a negative correlation between increases in business risk and $\left(\frac{I}{\bar{c}x - I}\right)$ suggesting that producers were balancing increased business risk with a decrease in financial risk.

Risk Efficiency Models

A number of risk efficiency approaches have been proposed in the academic literature. Risk efficiency models attempt to build an efficient frontier by maximizing expected returns subject to a risk measure. Perhaps the most common risk measure is the variance; however, recently risk measures such as the value at risk and to a lesser extent the conditional value at risk have gained prominence. The following section provides a general theoretical overview, highlights empirical applications and identifies strengths and weaknesses of each risk efficiency approach.

Expected Value Variance

Expected utility theory shows that risk adverse decision makers will be indifferent between the expected utility of a gamble and the utility of a certain guaranteed payoff. The difference between the expected rate of return for the gamble and rate of return for the certainty equivalent is denoted as the risk premium. The decision maker's

indifference between the expected utility of the gamble and the utility of the certainty equivalent can be expressed as

$$EU\left(\mu(\alpha)+\varepsilon\right)=U\left(\mu(\alpha)-\pi\right), \tag{2-6}$$

where EU is the expected utility operator, $\mu(\alpha)$ is the average portfolio return, ε is a normally distributed variable with a given variance, σ^2, and mean zero, U is the utility operator and π is the risk premium.

Equation (2-6) is a simplification of the method presented by Pratt (1964). The stochastic nature of the gamble is captured in the random variable ε. Expanding the left and right sides of expression (2-6) using a Taylor series expansion yields[1]

$$EU\left(\mu(\alpha)+\varepsilon\right)\approx EU\left(\mu(\alpha)\right)+E\left[\varepsilon U'\left(\mu(\alpha)\right)\right]+\frac{E\left[\varepsilon^2 U''\left(\mu(\alpha)\right)\right]}{2}, \tag{2-7}$$

and

$$U\left(\mu(\alpha)-\pi\right)\approx U\left(\mu(\alpha)\right)-\pi U'\left(\mu(\alpha)\right)^2, \tag{2-8}$$

where U' is the first derivative and U'' is the second derivative of the utility function with respect to $\mu(\alpha)$.

The expected value of the random variable ε is zero therefore the expression (2-7) collapses to

$$EU\left(\mu(\alpha)+\varepsilon\right)\approx U\left(\mu(\alpha)\right)+\frac{\sigma^2 U''\left(\mu(\alpha)\right)}{2}. \tag{2-9}$$

[1] The methodology employed here is closely related to the methodology used by John Pratt in his paper Risk aversion in the Small and Large.

Rearranging equation (2-9), such that $U(\mu(\alpha))$ is on the left hand side, substituting into equation (2-8) and then solving for the risk premium yields

$$\pi = \frac{\sigma^2 U''(\mu)}{2U'(\mu)}, \tag{2-10}$$

where π also equals μ minus the certainty equivalent μ_{ce} (Pratt 1964).

The risk premium can also be thought of as the difference between the expected return on the risky asset and the certain return on the risk free asset (Pratt 1964). Rewriting (2-10) as a function of the Arrow Pratt absolute risk aversion coefficient, R, μ and μ_{ce} and then solving for the certainty equivalent μ_{ce}, yields

$$\mu_{ce} = \mu(\alpha) - \frac{R(\mu(\alpha))}{2}(\sigma(\alpha))^2. \tag{2-11}$$

Equation (2-11) shows a clear indirect relationship between the variance and the certainty equivalent (Robison and Barry, 1987, p. 74). Replacing $\mu(\alpha)$ with the expected value of the portfolio the expected value variance problem can now be expressed in terms of the certainty equivalent.

$$Max\ E(\mu(\alpha)) - \frac{R(R(\mu(\alpha))}{2}(\sigma(\alpha))^2. \tag{2-12}$$

The tradeoff between risk and return can be mapped. Efficient portfolios maximize the certainty equivalent for a given α.

Empirical Applications of Expected Value-Variance

The theoretical framework proposed by Robison and Barry has been drawn upon many times in the agricultural economics literature. Kilmer, Andre, and Stevens (2001)

used a variation of the model presented by Robison and Barry to show that a negative relationship exists between fungicide and insecticide residues and the level of vertical integration for strawberries in Florida. A negative relationship was also found between insecticide residues and vertical integration for Florida tomatoes (Kilmer et al. 2001). The variant of the Robinson and Barry model used shows that a decrease in input quality variability can increase expected output. The increased expected output increases the certainty equivalent and corresponds with increases from the expected utility from profits.

Vertical coordination in the form of production contracts were investigated by Ross, Barry, and Gow (2001) for hog producers.[3] The authors document the risk shifting nature of production contracts and recognize that production contracts could be made more valuable to producers if they were more flexible because it would allow producers to optimize (2-12) in the short run. The compensation received by the producer from a production contract is modeled as a linear relationship consisting of both a fixed payment plus a variable incentive payment. The authors mention that the production contracts are not adjustable over the contract term. In formulating the argument the authors apply the expected value variance risk efficiency approach to the hog industry.

Producers have their choice of two lotteries. The risk free lottery generates a fixed payment to the hog producer while the risky lottery generates variable payments that are based on the producer's ability to capture incentive pay. Authors argue from a risk balancing standpoint that hog producers that pay down debt have reduced their financial risk. In order to keep the same level of total risk the producer must increase their level of

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[3] Ross, B., P. A. Barry, and H. Gow. "The Risk-Incentive Tradeoff : A Case of Illinois Hog Production Contracts." *Economics of Contracts in Agriculture Second Annual Workshop* (2002).

business risk. Current production contracts do not give a producer much flexibility to change the level of business risk.

Assuming producers had the flexibility to choose from a risky and risk free lottery, the authors show how producers can allocate production between the risky and risk free lottery by using the EV model. The authors show how a producer should allocate production and how that allocation should change when the producer's exposure to financial risk decreases. The authors conclude that production contracts should be designed to help growers shift risk.

Featherstone et al. (1990) used an EV model to quantify the gains from diversification to citrus producers. They found that for producers with moderate to high levels of risk aversion the cost of diversification outweighs the gains of diversification into strawberries, grapefruit and additional orange production. Moderately risk averse Florida's citrus growers were however able to reap gains from diversification by expanding into grapefruit production.

Advantages and Disadvantages to Using the Expected Value-Variance

The primary advantage of the expected value-variance (EV) approach is that it requires less information about the decision maker's preferences than expected utility.[4] In addition Robison and Barry (1987, p. 75) highlight the relative ease of using the EV approach and the "natural relationship between the concepts of risk and variability and the statistical concept of variance." Despite the advantages of the EV approach it cannot rank portfolios with same the expected value and variance but different higher moments.

[4] Using the Robison and Barry approach only the first and second derivative of the utility function are necessary.

Furthermore most of the sufficient conditions needed to establish consistency with expected utility have theoretical flaws or are not supported empirically.

Value at Risk and Conditional Value at Risk

According to Jorion (2001), the value at risk (VaR) allows decision makers to make statements such as we are α percent certain that wealth will exceed W^* dollars in the next N days. The value at risk for given probability α is denoted as an amount W^*. Value at risk attempts to quantify all aspects of risk in a single reportable number for a given confidence level.

Jorion (2001) defines the decision maker's end of period portfolio wealth as

$$W = W_0(1 - R), \tag{2-13}$$

where R is the rate of return over the period, and W_0 is the decision maker's initial wealth. The VaR criterion uses distribution functions to predict with α percent certainty that wealth will not drop below a critical level of wealth W^* in the next N days. The probability of the decision maker achieving less than the critical level of wealth W^* is calculated using the density of wealth function (Jorion 2001),

$$1 - \alpha = \int_{-\infty}^{W^*} f(W)dW. \tag{2-14}$$

The conditional value at risk (cVaR) for a given confidence level α then is defined as the expected wealth given that wealth drops below W^*. The cVaR can be represented as (Rockafellar and Uryasev 2002)

$$\frac{W\int_{-\infty}^{W^{*}} f(W)dW}{1-\alpha}. \tag{2-15}$$

Empirical Applications of Expected Value-Variance

Manfredo and Leuthold (2001) used the value at risk to predict losses in the cattle feeding margin using various VaR estimation techniques. The cattle feeding margin was defined as the difference between the revenue earned from the sale of fed cattle and expenditures on variable costs such as feed. The authors tested a number of different VaR parametric and non-parametric approximation approaches. The authors found that the riskmetric's VaR approximation approach and the historical simulation approach provided the best calibrated VaR measures for the 90, 95 and 99% confidence interval but noted that the results may not be indicative of future performance.

Bogentoft et al. (2001) uses the conditional value at risk as a constraint in a math programming model. The authors develop a framework for managing the assets and liabilities of a pension fund. The fund must collect funds from employees, invest funds and then pay funds out in the form of pension when the employees retire. The math programming problem that is set up is to minimize the total cost of funding the pension fund subject to the constraint that the funds liabilities will be funded with a very high probability. The authors estimate the conditional value a risk using the equation . The conditional value at risk is used instead of the value at risk because it is easier to optimize using linear programming.

A similar risk efficiency approach is used by Krokhmal et al. to derive the efficient frontier using the conditional value at risk. The case study included in the paper considers a portfolio of hedge funds. The efficient frontier was derived by maximizing

the expected returns of the portfolio subject to a cVaR constraint. The model was optimized by changing the relative weight of each hedge fund in the portfolio. Rockafellar and Uryasev (2000) note that the portfolios on the efficient frontier using the conditional value at risk are identical to the portfolios on the efficient frontier using the the variance risk measure when the return distribution is normally distributed.

Advantages and Disadvantages to Using Expected Value-VaR and Expected Value-cVaR

The value at risk communicates the risk facing a decision maker in a much clearer way than the variance or standard deviation. The clarity of reporting risk using VaR is perhaps one of the reasons why the Basel committee on Banking Supervision, the U.S. Federal Reserve Bank and the U.S. Securities and Exchange Commission use the VaR as the benchmark for risk reporting (Jorion 2001). There are however problems with using the VaR in a math programming model such as the EV approach. Rockafellar and Uryasev (2002) report that the VaR suffers from "being unstable and difficult to work with numerically when losses are not normally distributed. Furthermore the VaR does account for losses that exceed VaR. Rockafellar and Uryasev (2002) also write that the VaR is "incapable of distinguishing between situations where losses that are worse may be deemed only a little bit worse." Furthermore VaR is not a coherent measure of risk and is only consistent with expected utility maximization when portfolios can be ranked using first degree stochastic dominance.

The conditional value at risk is not able to communicate risk levels in the same conceptually pleasing way as VaR. While the cVaR has not been used as extensively as VaR it has begun to play a major role in the insurance industry. Furthermore cVaR does account for losses that exceed VaR and is therefore a more conservative risk measure

than VaR (Rockafellar and Uryasev 2002). The cVaR is easier to use in math programming than VaR because unlike VaR it is coherent risk measure even in the absense of normally distributed returns (Plug 2000). Furthermore cVaR is consistent with expected utility when portfolios can be ranked using second degree stochastic dominance, a more lenient condition than VaR. The conditions for consistency with coherency and expected utility maximization with be discussed in more detail in chapter 3.

Dairy Risk Mitigating Tools

Milk Futures

According to Leuthold (1989) a futures contract is a "transferable, legally binding, agreement to make or take deliver of a standardized amount of a commodity, or standardized minimum quality grade, during a specific month, under terms and conditions established by the federally designated contract market on which trading is conducted ."

The contract specifications for a class III milk contract specify that each futures contract should be valued at 2000 times the minimum class III USDA futures price. The USDA releases the minimum price for 100 pounds of class III milk with 3.5% butterfat on the fifth day of the succeeding month or on the previous business day if the fifth is not a business day. The last day of trading is the business day prior to the USDA release of the minimum class III milk price. New contracts are listed the day after the nearby contract expires for every month of the year. The trading prices for a given day are limited to one dollar more or less than the previous business day's settlement price. Open positions are cash settled to the minimum price released by the USDA.

Milk Options

There are two types of options. A call option gives the holder the right but not the obligation to buy the underlying asset at a given strike price at some point in the future. The put option gives the holder the right but not the obligation to sell the underlying asset at a given strike price a some point in the future. American options can be exercised at anytime up to the expiration date while European options must be exercised upon expiration (Hull 2000) .

Milk class III futures contracts serve as the underlying assets for class III and futures contracts. The class III milk options traded on the Chicago Mercantile Exchange are American style options.

Summary of What Was Learned

Agricultural producers face both business and financial risk. The sum of business and financial risk defines the producer's total risk exposure. The producer's response to financial risk given changes in business risk is know as the risk balancing hypothesis. The risk balancing hypothesis suggests that producers have a lexicographic preference for a given level of total risk. Empirical evidence suggests that when business risk increases producers try to reduce financial risk such that total risk remains constant. Currently a class III futures and class III put options are traded on the Chicago Mercantile Exchange. Both of these contracts can be used by producers to mitigate price risk. An interesting analysis that might be done is to determine the bounds of using futures and/or options to balance increased financial risk (the converse of the risk balancing hypothesis). However this analysis would require a formal definition of risk.

A number of different risk measures exist for measuring risk. Perhaps the most popular is the variance, however it suffers from serious theoretical drawbacks. The value

at risk clearly communicates the producers risk level however it is difficult to use in a math programming environment because it is not a globally convex function of the producer's hedging position. The conditional value at risk is similar to the value at risk and can be used easily in a math programming model because it is a globally convex function of the producer's hedging positions.

CHAPTER 3
COMPATIBILITY OF THE VARIANCE, VALUE AT RISK AND CONDITIONAL VALUE AT RISK WITH EXPECTED UTILITY MAXIMIZATION AND COHERENCY

Introduction

Chapter 3 will attempt to theoretically ground the variance, value at risk (VaR) and conditional value at risk (cVaR) risk measures. The first section of chapter three will lay the foundation for expected utility and highlight the necessary conditions required to achieve consistency for the variance, VaR and cVaR risk measures. The second section of chapter three will introduce the concept of coherency. Artsner (1997)first proposed four desirable properties of a risk measure. Risk measures that satisfy each of the four properties are deemed to be a coherent risk measure. The conditions required for the variance, VaR and cVaR to be coherent are presented in the second section of chapter three. The final section will summarized what has been learned and why the conditional value at risk was selected as the risk measure for this study.

Expected Utility

The development of expected utility has played a pivotal role in the application of economic theory to the real world. The traditional economic approach involves optimization within the context of certainty. The expected utility theorem is a stepping stone that enables us to get one step closer to a reality in which uncertain outcomes are the norm.

The discussion of expected utility will be broken down into five parts. The first part will contain a summary of Mas-Collell, Whinston, and Green's (1995) discussion of

lotteries as alternatives with objectively known probabilities. The second part of this section will include a discussion elaborating on the von Neumann and Morgenstern axioms. The third part of section one contains a proof of the expected utility hypothesis. The fourth section will discuss the certainty equivalent and how it can be used within the context of hedging. Finally the fifth section will highlight the conditions necessary to achieve consistency between expected utility, variance, VaR, and cVaR risk measures.

Simple Lotteries

The decision maker has a choice of alternatives with risky outcomes. Each element of the set of all possible outcomes is called a consequence denoted as c_i for i=1 to n. The set of all consequences is denoted in equation (3-1) by **C** (Mas-Colell et al. 1995) where

$$\mathbf{C} = (c_1, \ldots, c_n)\,. \tag{3-1}$$

The general case includes consequences as either goods or consumption bundles, however for this study the specific case of monetary consequences will be considered. Equation (3-2) represents the probability vector that corresponds to the consequence vector. Equation (3-3) constrains the sum of the probabilities to one. The equations are

$$\mathbf{P} = (p_1, \ldots, p_n) \tag{3-2}$$

and

$$\sum_{i=1}^{n} p_i = 1\,. \tag{3-3}$$

The simple lottery $L_s(\mathbf{P},\mathbf{C})$ is a function of a finite number of consequences and corresponding probabilities (Mas-Colell et al. 1995).

Compound Lotteries

The compound lottery is a function of simple lotteries. Represented in equation (3-4), the compound lottery includes simple lotteries as consequences, which have their own probability vector for the consequence. The compound lottery is represented as

$$L(\alpha_1, \alpha_2, L_1(\mathbf{P}', \mathbf{C}), L_2(\mathbf{P}'', \mathbf{C})), \tag{3-4}$$

where lottery one, $L_1(\mathbf{P}',\mathbf{C})$, is a function of $\mathbf{P}'$ and $\mathbf{C}$ for i=1 to n and lottery two, $L_2(\mathbf{P}'',\mathbf{C})$, is also a function of the same consequences **C**, but a different vector of probabilities, $\mathbf{P}''$. Both L_1 and L_2 are simple lotteries which are consequences of the compound lottery L. The probability of lottery L_1 occurring is equal to α_1 and the probability of lottery L_2 is equal to α_2. The sum of the probabilities equals one and can be represented mathematically as

$$\sum \alpha_i = 1 \quad 1 \geq \alpha_i \geq 0. \tag{3-5}$$

The decision maker will be indifferent to the compound lottery L and simple lottery L_s if the reduced probabilities are the same for the compound lottery as they are for the simple lottery[1] (Mas-Colell et al. 1995)

$$L_s(P, C). \tag{3-6}$$

The probabilities of the reduced form lottery equals the sum of the probability of each of the simple lotteries occurring times the probability of a particular consequence c_i for that lottery (Mas-Colell et al. 1995). The decision maker will be indifferent between the compound lottery L and the simple lottery L_3 if the reduced probability vector for the compound lottery is identical to P_3. This can be written as

[1] This is frequently referred to as the axiom of the reduction of compound lotteries.

$$L \sim L_s \Leftrightarrow L(\alpha_1 \mathbf{P}', \alpha_2 \mathbf{P}'' \ , \mathbf{C}) \equiv L_s(\mathbf{P}, \mathbf{C}) . \tag{3-7}$$

Von Neumann Morgenstern Axioms

The decision maker is indifferent to simple and compound lottery so long as the resulting probabilities and consequences are the same (Mas-Colell et al. 1995). That is to say that the reduced form of the compound lottery is the same as the simple lottery. Decision maker choice under certainty requires the assumption that the decision maker has complete and transitive and therefore rational preferences. In the following paragraph completeness and transitivity will be defined. The additional assumptions, continuity and independence, required for decision-making under uncertainty will also be defined.

Completeness

The completeness axiom assumes that for each lottery the decision maker is able to state preferences between lotteries, such that the decision maker is able to indicate whether they are indifferent ($\sim$), considers the lotteries to be at least as good as ($\sim\succ$), or strictly prefers one lottery to another lottery($\succ$) (Mas-Colell et al. 1995).

Transitivity

The completeness and transitive assumptions of the "at least as good as" operator implies that if lottery L_1 is at least as good as lottery L_2 and lottery L_2 is at least as good as lottery L_3 then lottery L_1 is at least as good as lottery L_3 as represented by

$$L_1 \sim\succ L_2 \sim\succ L_3 \Rightarrow L_1 \sim\succ L_3 . \tag{3-8}$$

Furthermore if lottery L_1 is strictly preferred to lottery L_2 and lottery L_2 is strictly preferred to lottery L_3 than lottery L_1 is strictly preferred to lottery L_3 as represented by

$$L_1 \succ L_2 \succ L_3 \Rightarrow L_1 \succ L_3 . \tag{3-9}$$

If the decision maker is indifferent between lottery L_1 and lottery L_2, the reflexive nature of the indifference operator ensures that the decision maker is also indifferent between lottery L_1 and lottery L_3 as represented by (Mas-Colell et al. 1995).

$$L_1 \sim L_2 \sim L_3 \Rightarrow L_1 \sim L_3. \qquad (3\text{-}10)$$

Continuity

The axiom of continuity rules out lexicographic preferences (Mas-Colell et al. 1995).[2] The decision maker always considers the tradeoff between the consequences of the preferred lottery L_1 and inferior lottery L_2. The decision maker will not discriminate against a lottery simply because of a miniscule probability of something bad occurring. Assume the decision maker prefers lottery L_1 to lottery L_2 and lottery L_2 to lottery L_3 then a probability, α, exists such that the decision maker is indifferent between the weighted linear combination of lotteries L_1 and L_3 and the lottery L_2. The decision maker's indifference is as

$$\alpha \in [0,1] : \alpha L_1 + (1-\alpha)L_3 \sim L_2. \qquad (3\text{-}11)$$

Independence

The fourth and perhaps the most controversial of the axioms necessary to derive the expected utility theorem is the axiom of independence.[3] The axiom of independence assumes that if a decision maker is faced with a compound lottery L^1 which has two simple lotteries L_1 and L_2, as its consequences, and compound lottery L^2 which has two

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[2] Lexicographic preferences are ordered preferences that are discontinuous and cannot be represented by a continuous utility function because the decision maker's utility is based solely on the quantity of the good that provides the most utility in the consumption bundle. There is no tradeoff in utility quantities of goods within the consumption bundle.

[3] This axiom is controversial because it has been shown to be violated frequently in empirical studies.

simple lotteries L_1,L_3 as its consequences then the decision maker is indifferent between lottery L^1 and L^2 only if (Mas-Colell et al. 1995)

$$L_2 \sim L_3 \Rightarrow L^1(\alpha, L_1, L_2) \sim L^2(\alpha', L_1, L_3)\,. \qquad (3\text{-}12)$$

Expected Utility Form

If the decision maker's preferences satisfy the rationality axioms and the axioms of continuity and independence, then there exists a utility function which maps the utility for each of those consequences in the expected utility form[4] (Mas-Colell et al. 1995). The expected utility of any consequence can be found by multiplying the probability of the consequence times the utility for that particular consequence. The expected utility for the lottery can be found by summing the expected utilities of all the consequences for that lottery.

Expected Utility Proof

The expected utility theorem proves that the expected utility form maximizes utility for all rational decision makers that satisfy the continuity and independence axiom. The theorem can be proven with the following proof (Mas-Colell et al. 1995). First assume that there is a best and worst consequence denoted as C^+ and C^- respectively. This implies that the best lottery, L^+, puts a 100 percent probability on consequence C^+ and the worst lottery, L^-, puts a 100 percent probability on consequence C^- respectively.

If L^+ is strictly preferred to (at least as good as) L^- and α equals a probability on the interval [0,1] then the utility of $\alpha(L^+)$ +(1-α)(L^-) is a(n) increasing (non decreasing)

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[4] If the utility of the compound lottery equals the summation of the weighted utilities of each of the simple consequential lotteries, then the utility of the compound lottery has an expected utility form.

$$U(\sum_{i=1}^{n} \alpha_i L_i) = \sum_{i=1}^{n} \alpha_i U(L_i)$$

function of α. This implies that the indifference curves between lotteries are linear and parallel in α space (Mas-Colell et al. 1995).

Any intermediary lottery L can be defined as a linear function of L^+ and L^- with parameter α. The decision maker's indifference to a simple lottery L_s and the compound lottery consisting of L^+ and L^- weighted by α and 1-α respectively is represented in expression by

$$L_s \sim \alpha L^+ + (1-\alpha)L^- . \tag{3-13}$$

Consider intermediate lotteries L and $L^{'}$. The expected utility proof hinges on showing that utility is a linear function of the probabilities employed by the simple lotteries L^+ and L^- such that

$$L \sim U(L)L^+ + (1-U(L))L^- \tag{3-14}$$

and

$$L' \sim U(L')L^+ + (1-U(L'))L^- . \tag{3-15}$$

The probability α is an index of utility $U(L)$ and $U(L')$ in expression (3-14) and (3-15) respectively.

In order to prove the expected utility form, it is necessary to prove that α is a linear function of the probabilities of the consequences in the best and worst lotteries. In order for the utility function of the compound lottery to be a linear function of the probabilities of the consequences of the simple lotteries L^+ and L^-, it must take the form of

$$U(\beta L + (1-\beta)L') \sim \beta U(L) + (1-\beta)U(L'), \tag{3-16}$$

where β is defined as a probability on the interval [0,1]. Given that utility can be indexed by α, substituting equations (3-14) and (3-15) into (3-16) yields

$$\begin{aligned} &U(B(L)+(1-B)L') \sim B(U(L)L^{+}+(1-U(L))L^{-})+ \\ &(1-B)(U(L')L^{+}+(1-U(L))L^{-}) \end{aligned}. \tag{3-17}$$

The right side of which is factored into a linear function of the best and worst lotteries[5]

$$\begin{aligned} &[BU(L)+(1-B)U(L')]L^{+}+ \\ &[(1-B)(1-U(L))+(1-B)(1-U(L'))]L^{-} \end{aligned}. \tag{3-18}$$

Equation (3-18) demonstrates that the utility is a linear function of the probabilities which forms the utility index $[BU(L)+(1-B)U(L')]$. This proves the expected utility theorem which states that the decision maker's utility from a compound lottery is simply the weighted average of the utility of the simple lotteries.

Certainty Equivalence

The expected utility form has several interesting implications. The certainty equivalent is the certain payoff an investor is willing to accept in lieu of a lottery. Consider the following example. Assume a simple lottery, L_s, with a Bernoulli payoff pays \$100 with probability equal to 50 % and pays \$10 with a probability equal to 50 %. Assume that the decision maker's preferences can be represented by the power utility function

$$U(C_t)=\frac{C_t^{1-\phi}}{1-\phi}, \tag{3-19}$$

[5] This is an application of the independence axiom.

where ϕ is a risk aversion coefficient assumed to equal 0.5 and C_i is a particular consequence (either 100 or 10 for this case). Applying the expected utility form to the lottery, the expected utility of $100 equals the probability 0.5 times the utility received from 100 dollars. Similarly the expected utility of $10 can be calculated as the probability 0.5 times the utility received from $10. The expected utility form shown previously in equation (3-18) proves the linearity between the utility index and the probability vector. Therefore, summing the expected utilities of each of the consequences, the expected utility of lottery L_1 can be discovered. The total utility expected for the decision maker equals 13.16, which is the sum of the expected utility of $100, which equals 10, and the expected utility of $10, which equals 3.16.

The utility of the certainty equivalent C_o equals the utility of the gamble (Mas-Colell et al. 1995)

$$U(C_0) = U\left(L_1(0.5, 0.5), (100, 10)\right). \tag{3-20}$$

The certainty equivalent then can be found by solving equation (3-19) for C_0 (a single consequence) when the left hand side is assumed to equal 13.16. When solved the value of the certainty equivalent equals 43.31.

Risk Premium

The risk premium π is the difference between $55, the expected value of lottery L_1, and $43.31, the certainty equivalent. If the certainty equivalent is less than the expected value of the lottery a positive risk premium is implied, which in turn implies that the decision maker is a risk averter.

Risk Aversion

The decision maker's level of risk aversion can be ascertained in a number of ways. The level of risk aversion can be extrapolated from the utility function or it can be represented by the risk premium paid by the decision makers.

Assume the existence of two decision makers D_1 and D_2, both of which have globally concave and monotonically increasing utility functions. The statement that D_1 is a least as risk avers as D_2 can be supported if D_2 is willing to accept any lottery D_1 is willing to accept. This occurs because D_2 has a lower risk premium. The decision maker with the higher risk premium will also have a utility function that is more concave. The Arrow-Pratt absolute risk aversion coefficient, defined as

$$R(c(L_1)) = -\frac{U^{''}(c(L_1))}{U^{'}(c(L_1))}, \tag{3-21}$$

where $R(c(L_1))$ is the Arrow Pratt absolute risk aversion coefficient applied to the certainty equivalent of lottery L_1; $U^{''}(c(L_1))$ is the second derivative of the decision maker's utility function applied to the certainty equivalent of lottery L_1 and $U^{'}(c(L_1))$ is the first derivative of the decision maker's utility function applied to the certainty equivalent of lottery L_1.

The Arrow Pratt risk aversion coefficient can be used to compare the absolute risk aversion of decision makers. If the absolute risk aversion coefficient for decision maker D_1 is greater than the absolute risk aversion coefficient for decision maker D_2 such that

$$R_{D_1}(c(L_1)) > R_{D_2}(c(L_1)), \tag{3-22}$$

then decision maker D_1 is more risk averse than decision maker D_2.

The risk aversion coefficient is always positive for risk averse decision makers, zero for risk-neutral decision makers and negative for risk loving decision makers. This translates into a positive risk premium for risk aversion decision makers, a zero risk premium for risk-neutral decision makers, and a negative risk premium for risk loving decision makers. Risk averse decision makers always have concave utility functions with respect to a certain consequence, risk-neutral decision maker always have utility functions that are linear with respect to a certain consequence and risk loving decision maker have utility functions that are convex with respect to a certain consequence (Mas-Colell et al. 1995).

Risk preferences can be classified by the first derivative with respect to a certain consequence. If the first derivative of the absolute risk aversion coefficient with respect to a certain consequence is positive (negative), the decision maker's preferences are classified as increasing absolute risk aversion (IARA), (decreasing in absolute risk aversion (DARA)). If the first derivative equals zero the decision maker has preferences that are consistent with constant absolute risk aversion (CARA).[6]

Expected Utility and Hedging

Traditionally hedging has been viewed as a risk mitigating tool. Keynes and Hicks described hedging as a tool used to transfer risk from risk averse hedgers to more risk tolerant speculators (Keynes 1930) (Hicks 1946). Producers know the historical distribution of prices for their particular commodity. They understand that there is a probability of achieving a very high price (the best consequence) and a probability of achieving a very low price (the worst consequence). Hedging allows producers to

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[6] A common utility function that is used to represent preferences that demonstrate CARA preferences is the negative exponential utility function, $U(C)=-\exp(-\phi*C)$.

theoretically lock in a fixed price (the certainty equivalent). Therefore in the traditional sense hedging is simply used to shift risk. Producers that are risk averse are willing to pay a premium for certain prices. Risk averse producers might opt to reduce their risk through hedging.

Framing the problem as an expected value-variance type problem allows the application of Robison and Barry's certainty equivalent methodology which parallels the approach used by Heifner (1973). The expected utility of a hedger with a position on the cash and futures market is given by

$$\begin{aligned} EU\left(C_0\right) &= X_s\left(E(P_{s1}) - P_{s0}\right) + X_f\left(E(P_{f1}) - P_{f0}\right) \\ &\quad - \frac{R}{2}\left(X_s^2\sigma_s^2 + X_f^2\sigma_f^2 + 2X_sX_f\sigma_{sf}\right) \end{aligned}, \tag{3-23}$$

where C_0 is the certainty equivalent, X_s is the long position on the cash market, $E(P_{s1}) - P_{s0}$ is the profits made on the cash market, X_f is the short position on the futures market[7], $E(P_{f1}) - P_{f0}$ is the profits made on the futures market, *R* is the Arrow Pratt absolute risk aversion coefficient and $X_s^2\sigma_s^2 + X_f^2\sigma_f^2 + 2X_sX_f\sigma_{sf}$ is the variance of the portfolio of spot and futures.

Assuming both the cash X_s and futures position X_f are endogenous the first order conditions can be calculated as

$$\frac{\partial EU\left(C_0\right)}{\partial X_s} = E(P_{s1}) - P_{s0} - RX_s\sigma_s^2 - RX_f\sigma_{sf} = 0 \tag{3-24}$$

and

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[7] Using this representation a short position is denoted as a negative value and a long position is denoted as a positive value.

$$\frac{\partial EU(C_0)}{\partial X_f} = E(P_{f1}) - P_{f0} - RX_f\sigma_f^2 - RX_s\sigma_{sf} = 0. \tag{3-25}$$

Solving equations (3-24) and (3-25) for X_s and X_f, the optimal cash and futures position are calculated as

$$X_s = \frac{\left(E(P_{s1}) - P_{s0}\right)\sigma_f^2 - \left(E(P_{f1}) - P_{f0}\right)\sigma_{sf}^2}{R\ \sigma_s^2\sigma_f^2 - R\ \sigma_{sf}^2} \tag{3-26}$$

and

$$X_f = \frac{\left(E(P_{f1}) - P_{f0}\right)\sigma_s^2 - \left(E(P_{s1}) - P_{s0}\right)\sigma_{sf}^2}{R\ \sigma_s^2\sigma_f^2 - R\sigma_{sf}^2}. \tag{3-27}$$

The expected utility maximizing ratio of cash to futures can be found without knowledge of the hedger's risk aversion coefficient R by taking the ratio of equations (3-26) and (3-27) which yields

$$\frac{X_s}{X_f} = \frac{\left(E(P_{s1}) - P_{s0}\right)\sigma_f^2 - \left(E(P_{f1}) - P_{f0}\right)\sigma_{sf}^2}{\left(E(P_{f1}) - P_{f0}\right)\sigma_s^2 - \left(E(P_{s1}) - P_{s0}\right)\sigma_{sf}^2}. \tag{3-28}$$

Equation (3-28) represents mathematically the ratio of the cash and futures position. Kahl (1983) compares the hedge ratio calculated by Heifner with the graphical representation by Ward (1971) and determines that the two methodologies are identical.

Expected Utility Maximizing Hedge Given a Cash Position

Peck derives the optimal futures position given that the cash market position is exogenous. The first order condition of the expected utility equation (3-23) is given by

$$\frac{\partial EU(C_0)}{\partial X_f} = E\left(P_{f1}\right) - P_{f0} - R\left(X_f\sigma_f^2 + X_s\sigma_{sf}\right) = 0, \tag{3-29}$$

which is a maximum because the second derivative $-2R\sigma_f^2$ must be negative. Solving for X_f yields the expected utility maximizing futures position (Peck, 1975)[8]

$$X_f = \frac{\left(E\left(P_{f1}\right) - P_{f0}\right) - R\left(X_s \sigma_{sf}\right)}{R\sigma_f^2}. \tag{3-30}$$

Unlike the case where both the cash and futures position is determined endogenously, when the cash position is fixed the producers hedge ratio is dependent on the risk aversion coefficient.

Each of the models presented above implicitly assumes that variance of returns can be used to as a risk measure. In actuality each of the above models implicitly assumes that the expected utility maximizing hedge ratio lies on the expected value-variance efficient frontier. The following section details the case where the literature has shown this to be the case.

Consistency of the Variance Risk Measure with Expected Utility

Quadratic Utility

The expected value-variance (EV) approach is frequently used because of its simplicity. In general the EV solution is not consistent with results from expected utility except under restrictive conditions. If the decision maker's preferences are defined by a quadratic utility function that is concave, then the utility function is defined by its first and second derivative. All higher derivatives are equal to zero; therefore the Taylor series approximation of the expected utility (defined by Pratt) of the gamble isn't an approximation, it is exact and the mean variance efficient set contains the expected utility maximizing portfolio (Kroll et al. 1984).

[8] Kahl says that while Peck doesn't use the same notation the two representations are identical.

Normally Distributed Returns

Distribution functions can be described using moment generating functions. The moment generating function for the normal density function is completely defined by its first two moments, the mean and variance respectively. When the decision maker has a negative exponential utility function and the return distribution is normally distributed then expected utility is a function of the expected return, variance and decision maker's risk aversion (Freund 1956).

Meyer's Location Scale

Meyer (1987) argues that neither the quadratic utility function nor the normally distributed rates of return criterion are acceptable from an empirical point of view. The quadratic utility function violates the monotonic assumptions imposed on the utility function by economic theory. Furthermore, lognormally distributed returns invalidates the normality assumption.

Meyer develops an alternative scenario. Meyer demonstrates that the cdf's are equal for distribution functions of choice sets that differ only with respect to a location and scale parameter such as

$$F_1(x)=F_2(\alpha+\beta x). \tag{3-31}$$

Consider stochastic variable Y and the stochastic normalized version variable X defined as

$$X=\frac{Y_t-\mu}{\sigma}. \tag{3-32}$$

Rearranging equation (3-32), the random variable equals

$$Y_t=\mu+\sigma X, \tag{3-33}$$

where μ is defined as the mean of Y and corresponds to the location variable α and σ is defined as the standard deviation and corresponds with the scale parameter β. The expected utility of variable Y is

$$EU = \int_a^b U(\mu + \sigma X) dF(X). \tag{3-34}$$

Therefore, since the expected utility can be written as a function of the mean and variance, consistency is established between the expected-value-variance and the expected utility (Meyer 1987). Meyer uses this fact to explain the identical findings of researchers using expected utility and expected value variance models even when the expected value-variance models fail to make the quadratic utility or normality assumptions necessary to achieve consistency. Meyer (1987) continues by arguing that the location and scale constraint is less restrictive than the quadratic and normality assumptions and that it leads to an outcome that is more theoretically palatable.

Consistency of Value at Risk and Conditional Value at Risk With Expected Utility Maximization

Consistency between a given risk measure and expected utility requires that the risk measure produce the same ordinal ranking of portfolios as expected utility maximization. The following section details when this can be achieved.

Normally Distributed Returns

When the underlying distribution is normally distributed the standard deviation is proportional to the value at risk and conditional value at risk. Therefore minimizing the variance[9], value at risk and conditional value at risk yields the same portfolios on the

[9] The variance is a monotonic transformation of the standard deviation. Therefore minimization of the standard deviation or variance subject to a return constraint yields the same efficient frontier.

efficient frontier (Rockafellar and Uryasev 2002). Utilizing Freund's result in conjunction with the result from Rockafellar and Uryasev implies that the efficient frontier created by minimizing the standard deviation, value at risk or conditional value at risk must contain the expected utility maximizing portfolio when the underlying return distribution is normally distributed and the decision maker has a negative exponential utility function.

First- and Second-Degree Stochastic Dominance

In practice however the return distribution is rarely normally distributed. This is particularly true in the dairy industry because of the truncated price distribution caused by the price-support program. When the distributional assumption made by Rockafellar and Uryasev is relaxed consistency can be achieved using stochastic dominance.

According to Yoshiba and Yamai (2001), VaR is consistent with expected utility when the cumulative distribution function of returns for the portfolios can be ranked using first degree-stochastic dominance. Therefore if lottery L_1 dominates lottery L_2 with first-degree stochastic dominance then the value at risk for lottery L_1 will also be greater than L_2 (Yoshiba and Yamai 2001). First-degree stochastic dominance however is very difficult to find in practice(Yoshiba and Yamai 2001) (Anderson et al. 1977). First-degree stochastic dominance only assumes that the decision maker's utility function is monotonically increasing. The economic literature usually assumes that the utility function is also concave. The concavity assumption as noted earlier is equivalent to assuming that decision makers are risk averse.

When the additional assumption of risk aversion is applied second-degree stochastic dominance can be utilized. The additional assumption gives second-degree stochastic dominance more discriminatory power than first-degree stochastic dominance.

However consistency with second order stochastic dominance and therefore expected utility is usually not achieved for VaR (Guthoff 1997). One exception to this generalization is the case when the underlying return distribution has an elliptical distribution. According to Yoshiba and Yamai when the lotteries can be ranked by second-degree stochastic dominance and the underlying distribution is elliptical then the ordinal ranking of lotteries by second-degree stochastic dominance is consistent with expected utility maximization. This however is problematic because most return distributions are not elliptical.

No distributional assumptions are required to achieve consistency with second-degree stochastic dominance when using the conditional value at risk. If two lotteries can be ranked using second-degree stochastic dominance then the same ordinal ranking must be given by the conditional value at risk. Therefore the conditional value at risk is consistent with expected utility maximization under more lenient conditions than the value at risk.

Consistency of Value at Risk and Conditional Value at Risk with Coherency

A second criteria for ranking risk measures is coherency. Artsner (1997) proposes four properties for coherency. He defines a coherent risk measure as a risk measure that is subadditive, positively homogenous, monotonic with respect to first-degree stochastic dominance, and translation invariance.

Subadditive and Positive Homogeneity

The property of subadditivity requires that diversification not create risk. Therefore when the risk measure ρ is applied individually to the losses from two assets x_1 and x_2, the combined risk must be less than or equal to the risk of the two assets. Mathematically Artzner (1997) represents this as

$$\rho(\lambda x_1 + (1-\lambda)x_2) \le \lambda\rho(x_1) + (1-\lambda)\rho(x_2), \tag{3-35}$$

where ρ is the risk measure operator and λ is a multiplier.

The property of subadditivity is limited by the property of positive homogeneity when there is no diversification effect. For example if the position is doubled for asset x_1 then there is no diversification effect and

$$\rho(\lambda x_1) = \lambda\rho(x_1) \tag{3-36}$$

for some constant λ.

Monotonic and Translation Invariance

The property of monotonicity assumes that if losses for x_1 are greater than losses for x_2 for every scenario then $\rho(x_1)$ must be greater than or equal to $\rho(x_2)$. The translation invariance axiom assumes that the addition of a constant deterministic variable κ affects the level of risk by the value of that constant such that

$$\rho(x_1 + \kappa) = \rho(x_1) + \kappa. \tag{3-37}$$

Several of the risk measures considered in the literature are not coherent. The standard deviation violates the property of monotonicity with respect to first-degree stochastic dominance in some cases. Although the standard deviation is monotonic with respect to first-degree stochastic dominance when losses (negative returns) are normally distributed, the empirical evidence suggests that losses are not typically normally distributed. The value at risk is not a coherent risk measure because it violates the property of subadditivity when losses are not normally distributed (Plug 2000). The conditional value at risk satisfies the subadditivity property without making any distributional assumptions. Furthermore cVaR is positively homogenous, translation

invariant and monotonic with respect to first and second-degree stochastic dominance and therefore coherent.

Using a Monte Carlo simulation, net firm income data was simulated for a dairy for November 2004. Artzner (1997) originally defined the coherency axioms for a loss distribution. The loss distribution equals the return distribution times negative one. For this study returns were simulated and the value at risk and conditional value at risk were calculated under various futures and options scenarios. Figure 3-1 illustrates various iso value at risk bands that can be achieved on various hedging strategies using futures and options. Consider portfolio A depicted on figure 3-1. Portfolio A consists of the cash position for a 1100 cow dairy that hedges 45% of its expected milk production with class III milk options for November 2004 and 74% of its expected milk production with class III milk futures for November 2004.

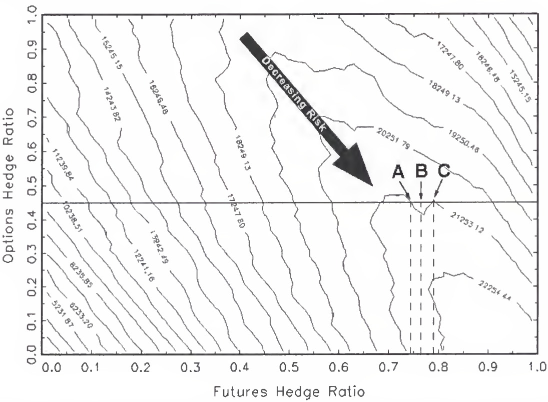

Figure 3-1. Iso VaR levels for various class III futures and options positions (November 2004)

The value at risk of portfolio A is equivalent to the value at risk of portfolio C which consists of a 45% hedge on expected milk production with class III milk options and a 78% hedge on expected milk production with class III milk futures. However if a portfolio consisting of 50% portfolio A and 50% portfolio C was constructed then the resulting portfolio would be portfolio B. The diversified portfolio is more risky than either of its components. This is a violation of the subaddititivity and positive homogeneity. The result is that iso conditional value at risk bands are not convex with respect to the decision makers futures hedge ratio. Similarly a vertical line could be drawn showing that the iso conditional value at risk contours are not convex with respect to the decision makers options hedge ratio.

Using the same simulated values iso conditional value at risk levels were generated for various futures and options positions in figure 3-2. When the conditional value at risk risk measure is used the iso cVaR contours are a globally convex function of both the futures and options hedge ratio.

Therefore for decision makers with preferences consistent with the von Neuman and Morgenstern axioms an efficient frontier can be constructed. When using the cVaR as a risk measure every point on the efficient frontier can be found by maximizing expected net firm income subject to a cVaR contraint.

Uryasev demonstrated that the portfolios used to generate the efficient frontier are the same when the conditional value at risk and variance risk measures are used. Furthermore while it is not possible to identify the actual expected utility maximizing portfolio it can be shown that the expected utility maximizing portfolio is located on the efficient frontier.

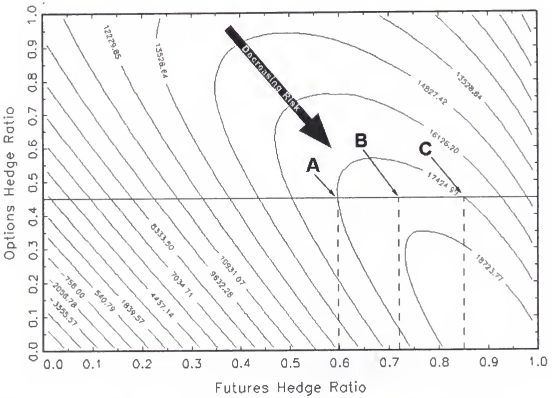

Figure 3-2. Iso cVaR Levels for Various Class III Futures and Options Positions (November 2004)

Summary of What Was Learned

The expected utility is the most theoretically grounded and strongest approach for decision making under uncertainty. The disadvantage to using the expected utility is that it requires cardinal knowledge of the decision maker's utility function. Risk efficiency approaches such as the expected value-variance, expected value-value at risk and expected value-conditional value at risk require much less information about the decision makers preferences and is therefore much easier to implement in practice.

The conditional value at risk is used as a risk measure instead of the value at risk and variance for three reasons. First of all, the conditional value at risk is consistent with expected utility under more lenient conditions than the value at risk and variance. Secondly, the conditional value at risk is a coherent risk measure under more lenient

conditions than the value at risk and variance. This yields a tangential benefit for future research. When expectations are introduced such that an expected gain or loss is expected from a given futures position it is easier to optimize a math program that utilizes the conditional value at risk as a risk measure than the value at risk. This result stems from the fact that the conditional value at risk satisfies Artzner's (1997) coherency assumptions of homogeneity and subadditivity. The assumptions of homogeneity and subadditivity imply that the risk measure is a globally convex function when the loss function is considered and a globally concave function when the return function is considered. The conditional value at risk is a globally concave function of the producers' futures and options position while the value at risk has multiple local minima. This result has been confirmed for the dairy industry empirically in figures 3-1 and 3-2.

CHAPTER 4
CONCEPTUAL MONTE CARLO MODEL FOR GENERATING MONTHLY NET INCOME SCENARIOS FOR A FLORIDA DAIRY PRODUCER

Depending on the producer's hedge ratio and leverage ratio the producer faces a given level of risk. For this study the risk level is defined as the producer's conditional value at risk. Calculation of the conditional value at risk is easiest to calculate using the delta normal method (Jorion 2001), however the delta normal method makes distributional assumptions about returns which are not appropriate for the US dairy markets.[1] The historical simulation approach uses actual data to formulate the producer's return distribution, but doesn't allow for the use of implied data and therefore it is difficult to account for time variation in price volatility (Jorion 2001). Furthermore recent structural changes in the dairy industry limits the amount of data that are available using the historical simulation approach.[2] The Monte Carlo approach to estimating the producer's conditional value at risk exposure allows for the use of a general return distribution (Jorion 2001) and therefore does not require as much data as the historical simulation approach. Furthermore the Monte Carlo approach allows for the use of implied data (Jorion 2001) and therefore does a good job of modeling time varying price volatility. The flexibility of the Monte Carlo simulation makes it the most powerful modeling approach; therefore it is well suited for markets such as dairy that are

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[1] The delta normal method assumes that the return distribution is normally distributed. This is not the case for dairy because the price floor causes the return distribution to be truncated.

[2] Federal milk marketing order reform, implemented in January 2000, consolidated the 31 federal marketing orders into 11 marketing orders, this structural change greatly impacted milk marketing in the United States.

characterized by atypical return distributions. For these reasons the Monte Carlo approach is the method that is used to generate the conditional value at risk estimates in this dissertation.

There are draw backs to using Monte Carlo simulation which should be noted. The primary drawback is model risk (Jorion 2001). The Monte Carlo approach relies on the stochastic process that is used to generate the return distribution. Therefore the integrity of the conditional value at risk estimates generated from the Monte Carlo simulation scenarios depends largely on model specification. The parameters used in the empirical model were estimated using various estimation techniques documented in the appendices.

Chapter 4 will define the underlying stochastic process for generating future net income scenarios. The variables and parameters used are defined in the text and are summarized at the end of the chapter. The final section of chapter 4 will discuss the methodology used to calculate the conditional value at risk.

Producer Revenue

Blend Price Revenue

The price that producers receive is based on a complex series of formulas.[3] The minimum price producers receive for delivered milk is called the blend price (Bailey 1997)[4], which equals the average of milk prices (weighted by utilization) in each of the

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[3] For the most up to date formulas used to price class I, II, III and IV milk visit http://www.aae.wisc.edu/future/front_cash_prices.htm.

[4] Bailey describes the basic approach used to calculate the blend price. The example used in his text is dated because it was written prior to order reform, but the basic methodology is consistent with the current calculation method.

four milk classes.[5] For this dissertation the mailbox price, or the estimated actual price received by producers at the farm gate, is assumed to equal the blend price plus the class I over order premium minus a correction for marketing deductions. Figure 4-1 illustrates the historical relationship between the Florida mailbox price reported by the USDA and the sum of the blend price and over order premium (3.12) minus a correction of 2.39.[6]

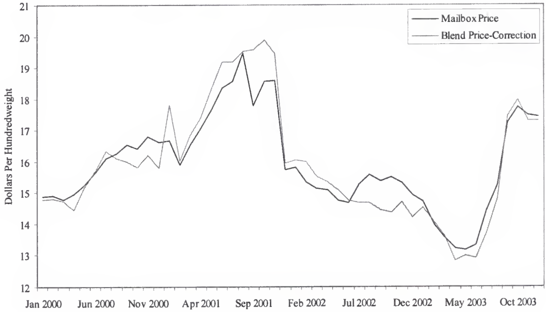

Figure 4-1. The historical relationship between the announced mailbox price and the summation of the blend price plus the class I over order premium minus 2.39

Simple linear regression reveals that 91.96% of the variation in the mailbox price can be accounted for by variation in the blend price.

Stochastic revenues earned through milk sales at the blend price were estimated using

$$B\tilde{P}R_t = \left(\tilde{C}_t^{I} + \tilde{C}_t^{II} + \tilde{C}_t^{III} + \tilde{C}_t^{IV}\right)C\tilde{W}T_t, \tag{4-38}$$

[5] The blend price is the average milk price in each of the four milk classes weighted by milk utilization in each class.

[6] The historical difference between the reported mailbox price and the summation of the blend price and over order premium was used as the correction.

where $\tilde{C}_t^{I}, \tilde{C}_t^{II}, \tilde{C}_t^{III}$, and $\tilde{C}_t^{IV}$ equal the stochastic weighted milk price received through milk sales to each of the respective classes for time period *t* and $C\tilde{W}T_t$ equals the stochastic milk production measured in hundredweights for time period *t*.

Milk Production

Stochastic monthly milk production equals

$$C\tilde{W}T_t = \frac{\left(P\tilde{P}C_t\right)\left(COW_t\right)\left(Days_t\right)}{100}, \tag{4-39}$$

where $P\tilde{P}C_t$ equals the stochastic milk production per cow per day for month *t*, $DAYS_i$ equals the number of days in month *i* and COW_t equals the average of cows milked during time *t*.

Milk production per cow per day tends to increase gradually over time. The gradual increase over time was modeled against the natural log of time where January 1987 was considered to be the base year. Data was readily available from 1987 through the end of 2003, furthermore it assumed that 17 years of production data will be enough to generate stochastic estimates of milk production per cow. Correcting for trend and seasonality the milk production per cow for a given month and year (≥ 1987) then could be predicted as

$$\begin{aligned} P\tilde{P}C_t &= 28.697 + 2.615 * Ln(EM) + \\ &6.922 * \sin\left(0.166EM * \pi\right) + 0.555 * \cos\left(0.166EM * \pi\right) + P\tilde{P}C_i, \end{aligned} \tag{4-40}$$

where 28.697 equals the regression intercept, 2.615 equals the regression coefficient used to correct for the long term gradual increase in milk production, 6.922 equals the regression sin coefficient, *EM* equals the number of elapsed months since January 1987, 0.555 equals the regression cosine coefficient and $P\tilde{P}C_t$ equals a stochastic residual for

month i. The residual varies by month. Random sampling of the residual was used to generate a stochastic estimate of milk production per cow. Appendix B highlights the methodology used to model stochastic milk production in more detail.

Milk Prices

Expression (4-38) also includes variables $\tilde{C}_t^{I}, \tilde{C}_t^{II}, \tilde{C}_t^{III}$, and $\tilde{C}_t^{IV}$ which previously were described as the stochastic revenue earned from milk sales in each of the respective milk classes. Milk sales in each of the milk classes are a function of the utilization in that market and the minimum FMMO price for utilization in that market. The following section details the methodology used to estimate a distribution of future prices for each class. Utilization is discussed after the discussion on milk prices in each milk class.

The minimum class I and II milk prices are based on the class III and class IV milk advanced milk prices (Manchester and Blayney 2001). Therefore, in order to define a stochastic process that accurately defines class I price movements a stochastic process must first be described for class III and class IV prices.

Class III and IV Prices. The distribution of future class III milk prices can be found using the binomial trees constructed in the following chapter. A 97 step binomial tree is constructed for a combination of unobservable futures and unobservable options. The 97 step tree exceeds the 30 minimum number of steps recommended by Hull (2002). The sum of the unobserved futures and unobserved options prices is calibrated such that it equals the price for the observed futures. Each step in the binomial tree corresponds with a given point in time. The binomial tree is drawn so that at step 97 the class III milk price is released. The class III milk price $\tilde{S}_t^{III}$ equals the class III futures price at expiration, $\tilde{F}_{T,T}^{III}$. Prices increase and decrease over time. Each up step through the tree is

the result of a Bernoulli trial when the random number sampled is less than p. The probability of a success p is a function of the number of steps in the binomial tree and the implied volatility of the futures contract. The methodology used to calculate p is described in detail in chapter 5.

For this study the distribution of class IV prices is found relative to class III prices. The difference between class IV and class III mover prices was recorded monthly on the day class I prices were released, v. The binomial tree was first used to find a class III at time v. The class IV price was then found relative to the class III price by sampling from the empirical distribution of differences between the prices of class IV and class III mover. This stochastic class IV effect will be denoted as $\tilde{E}_v^{IV}$. The class IV spot price then is estimated as

$$\tilde{S}_t^{IV} = \tilde{F}_{T,T}^{III} + \tilde{E}_v^{IV}. \tag{4-41}$$

Class I and II Prices. Class I and class II prices are modeled relative to the class III futures market. The following section contains background information related to these markets. The effective class one mover price for a class I hedger equals

$$\text{Effective Class I Mover} = \tilde{S}_t^{I} + \left(F_{t-l,T}^{III} - \tilde{F}_{v,T}^{III}\right), \tag{4-42}$$

where the class I spot mover for time t equals $\tilde{S}_t^{I}$ and $F_{t-l,T}^{III}$ equals the price at time t-l on a futures contract expiring at time T, and $\tilde{F}_{v,T}^{III}$ is the price on the same contract at the time class I prices are released, v. Equation (4-42) can then be rearranged to show that the producer's ability to lock in a price is a function of the closing basis. Figure 4-2 illustrates the historical class III enter price versus the effective class I mover when the hedge is set to 6 (l=6) months prior to expiration. The number of months a contract

remains on the board has varied over time. Selecting a lag of 6 months allowed for the inclusion of all months from January 1999 to December 2003 while maintaining a respectable hedging interval.

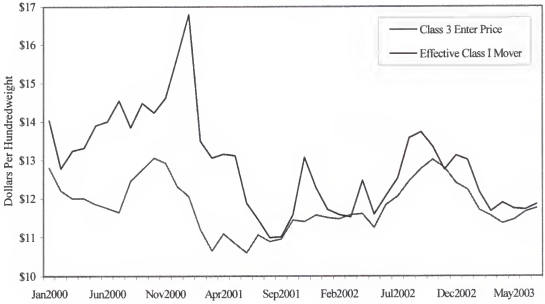

Figure 4-2. The class III enter price versus the effective class I mover

The difference between the effective class I mover and the class III enter price equals the closing basis. The closing basis can be decomposed into the mover and acceleration effect represented in figure 4-3.

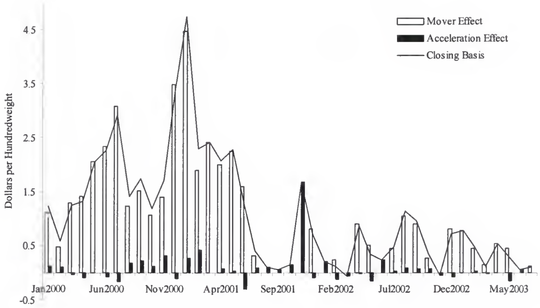

Figure 4-3. Decomposing the closing basis into the acceleration and mover effect

Class I prices are based on the maximum of the class III and class IV advanced prices. If the producer locks in the class III price and the advanced class IV price ends up being the mover then the there exists positive pressure on the positive closing basis which works towards the producers benefit. For this study this is referred to as the "mover effect." Historically linear regression reveals that 94.77% of the variation in the closing basis can be explained by the "mover effect." The remaining variation is caused by the "acceleration effect."

Advanced prices are based on two NASS weekly survey periods instead of four or five NASS weekly survey periods. Furthermore, the advanced pricing factors (survey prices based on 2 weeks of a NASS survey data for manufactured dairy products) are released on the Friday before the 23rd of the month unless this date is a Friday.[7] Class III and class IV prices are released on the Friday prior to the 5th of the following month

[7] http://www.ams.usda.gov/dyfmos/mib/prc_rls_date_03.pdf

unless this date is a Friday. The different release dates coupled with the different survey period can cause current class III price to diverge from the advanced class III price. For this study divergence that is due to these factors is called the "acceleration effect."

While the "mover effect" only works towards the producer's advantage the "acceleration effect" can work towards the producers detriment. Therefore any hedging strategy that is employed should attempt to minimize the "acceleration effect." The acceleration effect is minimized when producers use futures lagged by one month (i.e. hedge November milk with an October futures contract) and round turn their position on the day that the class I mover is released. This result will be discussed in more detail in chapter 6. The "acceleration effect," is modeled as a stochastic value sampled from an empirical distribution. The class I mover then is estimated as

$$\tilde{S}_t^I = \max(\tilde{F}_{v,T}^{III} + \tilde{E}_v^{IV}, \tilde{F}_{v,T}^{III}) + C_D^I + \tilde{E}_v^A \ , \tag{4-43}$$

where the price of class III futures on the day the class I mover is released equals $\tilde{F}_{v,T}^{III}$, the class IV effect equals $\tilde{E}_v^{IV}$, the fixed class I differential equals C_D^I and the acceleration effect is denoted as $\tilde{E}_v^A$. Class II prices are estimated to equal

$$\tilde{S}_t^{II} = \tilde{F}_{v,T}^{III} + \tilde{E}_v^{IV} + \tilde{E}_v^A \, . \tag{4-44}$$

Milk Utilization

Two multiple linear regressions with restrictions were used to describe changes in milk utilization over time. Federal milk order reform in January of 2000 resulted in a structural change for the Florida Dairy Industry. For this reason the analysis of milk utilizations presented in appendix A is limited to data from January 2000 to December 2003.

The regressions presented in appendix A attempt to do two things. First of all, tastes and preferences of the American public has changed over time. Changing tastes and preferences have impacted how milk is utilized in the United States. American tastes and preferences are a function of changing societal norms, a changing family structure and immigration. [8] A trend variable was included to measure the change in utilization over time in each of the four milk classes. It was expected that milk utilization would decrease in the class I market and increase in the class III market.

The second objective was to determine the seasonality of milk utilization. Milk production in Florida is seasonal, while milk consumption is relatively stable. For this reason a dummy variable was included to measure the seasonal effects on utilization when milk production was lowest. It was expected that milk utilization in the class I market would increase during the summer months. This was expected because of the relatively inelasticity of the class I market. Reduced milk production during the summer months means that the varying utilizations need to compete for the milk. Given the nature of the class I fluid market it is not surprising that most of the research has shown that the bulky, perishable class I product is more inelastic than the utilizations in the other markets. Dummy variables were included to capture the change in milk utilization due to season.

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[8] Fluid milk consumption is decreasing over time. This trend has been fueled by changes in the typical American family. Immigration has reduced the relative proportion of Americans with European roots. Dairy products have traditionally been a staple in the diets of Caucasian people while they have played a lesser role in the diets of individuals from different cultures. Furthermore people eat out more today than in the past. People who come from broken homes are more likely to eat out than their counter parts that come from traditional homes. As more households find it necessary to have two incomes the increasing trend to eat out is only reinforced. These trends have impacted the dairy industry in a number of ways. First of all milk consumers for some reason prefer to be "closet drinkers." While for many people it is common to enjoy a glass of milk for dinner when dinner is consumed in the privacy of the home the same as not true when these same individuals eat out. Consumers that eat out are less likely to consume milk as a beverage than there counterparts that consume milk at home, they are however more likely to consume milk in the form of cheese.

The trend variables were modeled for each market separately. Intercept terms were included for each market for each quarter. The quarterly class intercepts were used as explanatory variables to explain seasonality in milk utilization. Three restrictions were imposed on the regressions. The first restriction was that the sum of the intercept terms must sum to one. The second restriction restricted the sum of the trend variables to equal zero while the third restriction restricted the sum of the dummy variables for each quarter to equal zero. The restrictions were imposed in order to ensure that the utilization predictions for a given month summed to 100%.

The stochastic utilizations for the class I, class III and class IV markets respectively corrected for trend and month are given by

$$\tilde{U}_t^I = Q_i^I - 0.00077\left(EM_2\right) + U\tilde{T}IL_i^I, \tag{4-45}$$

$$\tilde{U}_t^{III} = Q_i^{III} + 0.000317\left(EM_2\right) + U\tilde{T}IL_i^{III}, \tag{4-46}$$

and

$$\tilde{U}_t^{IV} = Q_i^{IV} + 0.000104\left(EM_2\right) + U\tilde{T}IL_i^{IV}. \tag{4-47}$$

The Q_{1-4}^{I-IV} terms equal an intercept that varies by class and quarter (i.e. when i=1,2, or 3 the quarter equals 1), the monthly trend in each respective class is captured by the regression coefficient of EM_2, the number of elapsed months since January 2000, and $U\tilde{T}IL_{1-4}^{I-IV}$ equals a stochastic residual which varies by month and class. The stochastic estimates of utilization are found by randomly sampling from the residuals for each respective class and quarter. The Class II utilization was estimated as one minus the sum of the corrected utilizations in each of the other markets.

The price (weighted by utilization) per hundredweight earned in each respective milk class at FMMO minimum prices equals

$$\tilde{C}_t^I = \left(\tilde{S}_t^I\right)\tilde{U}_t^I, \tag{4-48}$$

$$\tilde{C}_t^{II} = \tilde{S}_t^{II}\left(1 - \tilde{U}_t^I - \tilde{U}_t^{III} - \tilde{U}_t^{IV}\right), \tag{4-49}$$

$$\tilde{C}_t^{III} = \left(\tilde{S}_t^{III}\right)\tilde{U}_t^{III} \tag{4-50}$$

and

$$\tilde{C}_t^{IV} = \left(\tilde{S}_t^{IV}\right)\tilde{U}_t^{IV}, \tag{4-51}$$

where the $\tilde{S}^{I-IV}$ terms equal the spot prices in each of four market classes, the $\tilde{U}^{I,III,IV}$ terms equal the stochastic utilization in each of the four classes. The sum of (4-48), (4-49),(4-50), and (4-51) equals the blend price, which is the minimum price that producers can be paid for their milk.

Stochastic Over order Premium

Producers are free to negotiate for prices above the blend price. Premiums received above the blend price are primarily a function of the cooperative's bargaining power. The producer's bargaining power varies seasonally. During the hot summer months when milk production is low cooperatives, which are bound by full service contracts, must import milk from out of state in order to meet processor needs. Milk is bulky and expensive to ship. Therefore during the months when the cooperative has to import milk the producer's over order premium is typically higher. A recently conducted study by Prasertsri (2002) assumes the cooperative and the processor benefit by cooperating with each other. If bargaining breaks down the processor must seek another seller and pay a higher price. Prasertsri assumes that the higher price paid by the processor equals the weighted average of milk prices in four surplus areas (Baltimore, Detroit, Kansas City,

and Philadephia). Furthermore if bargaining breaks down the cooperative must seek another buyer and will receive a discounted price equal to the price of manufacturing grade milk. The bargaining model proposed by Prasertsri (2002) finds the over order premium relative to the class I price as a function of the cooperative's and processor's bargaining power, the cooperative's and processor's risk aversion levels and the alternative price paid by the processor and received by the cooperative if bargaining breaks down.

While the over order premium varies significantly from month to month as a result of bargaining power, for this study the overorder premium was estimated simply as 3.12 for simplicity (using historical over order premium data from January 2000 to December 2003. Revenue earned from the over order premium equals

$$O\tilde{V}R_t = \tilde{U}_t^I * 3.12 * C\tilde{W}T_t, \tag{4-52}$$

where $\tilde{U}_t^I$ equals the previously discussed class I milk utilization..

Milk Income Loss Contract (MILC)

Producer revenues are also influenced by the milk income loss contract (MILC) program. The MILC pays 45% of the difference between the Boston class I price and \$16.94 for milk production up to 2,400,000 pounds of milk (Blayney 2002). The direct payments from the MILC program are similar to a 45% put option hedge on 2,400,000 pounds with a strike price equal to \$13.69 on the class I mover.[9] For larger Florida dairy farms the counter cyclical benefits of MILC are limited because milk production quickly exceeds the 2,400,000 pounds covered by MILC. The estimated production covered by milk for time t equals

[9] The difference between 16.94 and 13.69 is the class I differential for the Boston Market.

$$M\tilde{ILCP}_t = \max\left[\min\left(\left(2,400,000 - \sum_{z=t-s}^{t-1} E\left(C\tilde{WT}_z\right) + C\tilde{WT}_t\right), C\tilde{WT}_t\right), 0\right], \quad (4\text{-}53)$$

where s is an index representing the number of months prior to the hedge that the producer enrolled in MILC. The revenue earned from MILC for a given time period equals

$$M\tilde{ILCR}_t = MR * M\tilde{ILCP}_t * \max\left(13.69 - \left(\max\left(\tilde{F}_{v,T}^{III} + \tilde{E}_v^{IV}, \tilde{F}_{v,T}^{III}\right) + \tilde{E}_v^{A}\right)\right) * C\tilde{WT}_t, \quad (4\text{-}54)$$

where MR equals the ratio of (4-53) to expected milk production for a given time t times 0.45.[10]

Other Revenue Sources

Producers need to pay marketing fees. The cost to the producer for these fees equals

$$C\tilde{O}R_t = CO * C\tilde{WT}_t, \quad (4\text{-}55)$$

where CO equals the 2.39 correction highlighted in figure 4-1. Producers also earn revenue from a miscellaneous source such as cattle sales. Revenue earned from these sources is included in the model as

$$M\tilde{IS}R_t = MIS * C\tilde{WT}_t, \quad (4\text{-}56)$$

where MIS is calculated to equal 0.81 using data from 2001 DBAP data.

[10] For up to date information on the Milk Income Loss Contract Program visit http://www.fsa.usda.gov/dafp/psd/MILC.htm

The revenue earned from the producers futures position equals

$$\tilde{F}R_t = HF_t^{III} * E\left[C\tilde{W}T_t\right]\left[F_{t-l,T}^{III} - \tilde{F}_{v,T}^{III}\right], \quad (4\text{-}57)$$

where HF_t^{III} is a decision variable that equals the ratio of milk hedged using futures to milk produced, l equals the number of months prior to production that the hedge is laid and v equals the day that the class I mover is released. Similarly the net revenue earned from the producers options position equals

$$\tilde{O}R_t = HO_t^{III} * E\left[C\tilde{W}T_t\right]\left[\tilde{O}_{v,T}^{III} - O_{t-6,T}^{III}\right] , \quad (4\text{-}58)$$

where HO_t^{III} is a decision variable that equals the ratio of milk hedged using options to milk produced, $O_{t-6,T}^{III}$ is the value of the option at time t-6 on a futures contract that will expire at T and $\tilde{O}_{v,T}^{III}$ is the value of that same option when it is sold on the day the class I futures price is announced.

Producer Expenses

Production Expenses

The cost to producers to produce one hundredweight of milk was calculated as a function of the number of cows milked, the producer's production per cow and the producer's geographic location. These estimates were determined econometrically. The details of the methods used to calculate these estimates can be found in appendix C.

The total cost to producers equals

$$E\tilde{X}P_t = \left(16.54\text{-}0.184\left(P\tilde{P}C_t - 46.93\right) + 0.003\left(P\tilde{P}C_t - 46.93\right)^2 + NORTH\right)C\tilde{W}T_t, \quad (4\text{-}59)$$

where 16.54 is the cost to produce a hundred weight of milk in Florida's central region on a dairy with 1230 (the average size dairy for DBAP participants) dairy cows averaging 46.93 pounds of milk (the average milk production for DBAP participants), -0.184 is a

parameter estimate compensating for the increased cost realized by dairies with milk production less than the average DBAP average milk production 46.93, 0.003 is a parameter that limits the gains from increased milk production per cow and *NORTH* is a parameter calculated which indicates the increased cost realized by dairy producers the Northern part of Florida.

Interest Expenses

The producer also realizes expenses from debt financing. The interest expense realized by the producer equals

$$INT_t = r_t\left(\frac{Days_t}{365}\right)d * 3836 * COW_t, \tag{4-60}$$

where r_t equals the producers interest rate for time *t*, *d* equals the producer's debt to asset ratio for the same time period and 3836 equals the amount of assets employed per cow, which was estimated using 2001 DBAP data.

Producer Net Income

The producer's net firm income for a given month can then be summarized as

$$NFI_t = B\tilde{P}R_t + O\tilde{V}R_t + MI\tilde{L}CR_t + M\tilde{I}SR_t + C\tilde{O}R_t + \tilde{F}R_t + \tilde{O}R_t - E\tilde{X}P_t - INT_t. \tag{4-61}$$

The Monte Carlo simulation is used to simulate net income scenarios. In the process of generating scenarios stochastic variables were simulated such that the historical correlations in table 4-1 were enforced.

Table 4-1. Historical correlations between stochastic variables

	$U\tilde{T}IL_i^I$	$U\tilde{T}IL_i^{III}$	$U\tilde{T}IL_i^{IV}$	$\tilde{F}_{T,T}^{III}$	$\tilde{F}_{v,T}^{III}$	$\tilde{E}_v^A$	$\tilde{E}_v^{IV}$	$P\tilde{P}C_t$
$U\tilde{T}IL_i^I$	1.00	-0.82	-0.86	0.34	0.34	0.13	0.03	-0.36
$U\tilde{T}IL_i^{III}$		1.00	0.54	-0.48	-0.49	0.07	0.19	0.41
$U\tilde{T}IL_i^{IV}$			1.00	-0.16	-0.14	-0.14	-0.06	0.23
$\tilde{F}_{T,T}^{III}$				1.00	0.00	-0.32	-0.43	-0.38
$\tilde{F}_{v,T}^{III}$					1.00	0.02	-0.60	-0.42
$\tilde{E}_v^A$						1.00	0.00	-0.01
$\tilde{E}_v^{IV}$							1.00	0.37
$P\tilde{P}C_t$								1.00

Optimization Problem

For this study it is assumed that the futures market is unbiased. Therefore producers are not able to change their expected returns using futures and options contracts. Producers are however to adjust their level of risk. For this study the conditional value at risk at the 10% confidence level was used to calculate the producers risk exposure. The producers risk level for a given futures and/or options position was calculated by taking the average of the worst 10% of net firm income simulations.

CHAPTER 5
RISK-NEUTRAL VALUATION OF AMERICAN CLASS III MILK OPTIONS USING BINOMIAL TREES AND THE GENERATION OF STOCHASTIC CLASS III PRICES

The binomial trees constructed for this study serve a dual purpose. First of all, the binomial trees enable the valuation of American style options trading on class III milk futures. Secondly the binomial trees presented in this chapter are used to predict the future distribution of futures and cash prices.

Risk-Neutral Valuation

Binomial trees are constructed based on the assumption that a security can be priced "as if" the world's investors had risk-neutral preferences (Hull 2000). Risk-neutral valuation implies two things. First of all risk-neutral valuation assumes that the expected return on a portfolio of all securities traded equals the risk free rate. Secondly risk-neutral valuation implies that future cash flows can by valued by discounting at the risk free rate. After creating a risk free portfolio no arbitrage arguments can be used to value individual securities.

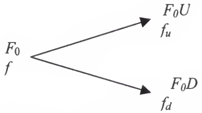

Figure 5-1. One Period Binomial Tree

Consider figure 5-1, which shows the price of long futures and a short call option at two different time periods. Initially the observed futures price equals F_0. The futures

price either increases in the following period to F_0U or prices decrease to F_0D where U and D are the proportional increases and decreases in the initial futures price (Hull 2000). The call option on futures has an initial value of f and takes a terminal value of f_u if the underlying futures increases and a terminal value of f_d if the underlying futures contract decreases. A portfolio consisting of Δ long futures and one short call option is created such that regardless of price movements in the underlying futures the terminal value of the portfolio remains constant.[1] Since the portfolio is insensitive to the state of the world (no risk) the portfolio should earn the risk free rate. This portfolio can be found by finding Δ such that

$$F_0U\Delta - f_u = F_0D\Delta - f_d \tag{5-1}$$

where

$$\Delta = \frac{f_u - f_d}{F_0U - F_0D}. \tag{5-2}$$

The present value of the portfolio equals

$$\left(\left(F_0U\Delta - F_0\right) - f_u\right)\exp(-r\delta t), \tag{5-3}$$

where r equals the risk free rate and δt equals the amount of time that will elapse until the options expires (Hull, 2000).[2] Another expression for the present value of the portfolio equals the value of the call option today f, the cost of setting up the portfolio. Under no arbitrage

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[1] This is frequently referred to as delta neutral hedging.

[2] Time here is expresses as a percentage of a year. For example if the option is sold on January 1, 2003 and is set to expire on July 1, 2003 δt would equal 0 .5.

$$f = F_0\Delta - \left(\left(F_0U\Delta - F_0\right) - f_u\right)\exp(-r\delta t). \tag{5-4}$$

Substituting for Δ, canceling redundant terms, factoring out the terminal option values and multiplying both sides by a negative one yields the current price of the option

$$f = (1-p)f_d\exp(-r\delta t) + pf_u\exp(-r\delta t), \tag{5-5}$$

where the probability of an up movement is denoted as (Hull 2000)

$$p = \frac{1-D}{U-D}. \tag{5-6}$$

Relationship between *U*, *D*, and Volatility

The up and down movements used in this paper are calculated following the approach given by Cox, Ross and Rubenstein (1979). According to Hull the volatility of futures prices σ is defined so that $\sqrt{\sigma^2\delta t}$ is the standard deviation of the return on the stock price for a short period of time δt (2000). The variance of the returns on the futures contract equals

$$\sigma^2\delta t = pU^2 + (1-p)D^2 - \left(pU + (1-p)D\right)^2. \tag{5-7}$$

One method that has been used extensively in the literature is to ignore δt terms of order two and higher. Based on this assumption U and D can be found as a function of the volatility of the underlying security. Solving for the proportional up and down movements yields (Hull 2002)

$$\begin{aligned} U &= \exp\left(\sigma\sqrt{\delta t}\right) \\ D &= \exp\left(-\sigma\sqrt{\delta t}\right) \end{aligned}. \tag{5-8}$$

The up and down movements are expressed as a function of the volatility and the length of the time step (Hull 2000).

Implied Volatilities

Historical price volatility can be measured by taking the standard deviation of the natural logarithms of successive prices (Hull 2000). For the purposes of this study this methodology is inadequate. One of the objectives of this study is to formulate hedging strategies that will enable dairy producers to balance business and financial risk. It is difficult for producers to reduce their financial risk in the short run because financial risk is largely a function of the producer's capital structure and the capital structure of privately held agricultural firms is relatively inflexible in the short run. Therefore in order to formulate a hedging strategy that will allow producers to react to increased price volatility it is first necessary to predict future price volatility. Historical measures of price volatility do not incorporate structural market changes into calculations until markets have already become volatile.

Two methods are typically used to forecast future volatility. An econometric approach that is commonly utilized in the economic literature utilizes GARCH modeling. Another approach that is frequently used is the implied volatility method. Implied volatility approaches imply the volatility of the underlying asset using option premium data. The implied volatility approach is arguably the market's best estimate of the volatility of the underlying asset prices. Manfredo et al. (2001) evaluated the performance of alternative volatility forecasts for fed cattle, feeder cattle, and corn cash price returns. Time series (e.g. GARCH), implied volatility from options on future contracts, and composite specifications were evaluated. Their results indicate that no single method of volatility forecasting provides superior accuracy. For purposes of this research the implied volatility measure will be used because of the computational ease of using implied volatilities. The ability to quickly calculate volatilities using readily

available market data that is freely accessible to anybody with an internet connection greatly increases the probability that practitioners will adopt recommendations generated from this research.

Two different methods can be used to imply the volatility of futures price when European options are traded. The Black-Scholes option-pricing model can be used to imply volatilities for futures by finding the volatility measure that yields the correct options price. Another approach that is frequently used is the binomial tree method. The binomial tree method requires more computational resources but is more flexible than the Black-Scholes approach. The flexibility of the binomial method allows it to be used to price American as well as European style options. European options on class III milk futures are not traded. American style options are the only type of option that currently trades on class III milk futures. Therefore for this study implied volatilities used to construct the binomial trees will themselves be calculated using the binomial tree approach. The volatilities are then used to calculate the actual U and D for the binomial tree used to value American options.

Valuation of American and European Options

Once the volatility of the underlying futures has been implied and the up, down and corresponding risk-neutral probabilities have been calculated the binomial tree can be constructed. Binomial trees can be used to value American and European style options. The binomial tree is drawn the same regardless of the risk free rate and regardless of whether the option being valued is American or European. The trees in conjunction with the risk free rate can be used to forecast the value of American or European options at any given future point in time. A valuation procedure for American and European style options differ slightly because embedded in the American option is an additional option

that allows for early exercise. As a result the American style option is always worth at least as much as its European counterpart.

There are some instances when the value of the American style option might equal the value of a European style option. These are instances when it is never optimal to exercise the option early. One example of such is the case of a American call option on a non dividend paying stock (Hull 2000).[3] While it can be shown that early exercise is never optimal for an American call option on a non-dividend paying stock the same is not true for an American put option on a non-dividend paying stock.[4] The value of the put option is truncated because prices cannot fall below zero. Therefore when the value of the put is deep in the money, sometimes the investor is better off harvesting gains on the put through early exercise. Hull (2000) illustrates this point by considering an American put option with a strike price of $10 and an underlying security price near $0. Prices are truncated at zero, therefore the put option cannot become much more valuable. Early exercise of the put option on a non-dividend paying stock is therefore optimal for deep in the money puts, because gains are not likely to increase and because once the gains from exercise are realized they can be reinvested at the risk free rate (Hull 2000).

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[3] It is never optimal to exercise an American call option on a non dividend paying stock early. An investor that holds the call option is better off exercising the option at expiration because it allows the investor to delay paying the strike price until a later date. By waiting to pay the strike price the strike price can be invested at the risk free rate until the option expires. Furthermore while holding the option the investor is protected against adverse movements in the underlying stock price Hull, J. *Options, Futures, and Other Derivatives.* 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2000.

[4] It is sometimes optimal to exercise an American call or put option on a futures contract early because a futures contract is similar to a stock that pays a continuous dividend that equals the risk free rate Ibid.. Another argument might be that futures contracts are theoretically free. The investor who exercises a call option against a futures contract does not pay the strike price up front. An investor that exercises an American call option against a stock early does pay the strike price. Therefore the argument presented earlier that holders of American call option on a non dividend paying should invest the value of the strike price and earn the risk free instead of exercising the option early doesn't hold when the underlying asset is a futures contract.

The price of an option can be though of as (Hull 2000)

$$\text{Option Price=Intrinsic Value +Time Value}. \tag{5-9}$$

The intrinsic value of a European or American call option equals

$$C^{iv}_{t,t+k} = \max(F_{t,t+k} - X, 0) \tag{5-10}$$

and the intrinsic value of value of a European or American put option equals

$$P^{iv}_{t,t+k} = \max(X - F_{t,t+k}, 0), \tag{5-11}$$

where X is the exercise price of the put or the call and $F_{t,t+k}$ is the current price on the underlying futures with expiration in period $t+k$. The price of an American option must be at least as great as its intrinsic value. The actual value of the option minus its intrinsic value equals its time value. Therefore when an investor exercises an option early the gain that is realized equals the intrinsic value and the time value is forfeited.[5] The investor that chooses to exercise the American put option on a non-dividend paying stock must determine whether the gains from early exercise exceed the time value of the option that will be forfeited by early exercise.

When determining whether or not to exercise early holders of American put and call options must determine whether the gains of early exercise exceed the time value of the option that will be forfeited. American style options require that the value of the option be calculated at each of the node times. For an American option the value equals the maximum of its discounted expected value and its value from early exercise. Mathematically the value of an American put option equals

[5] The time value of an option can be thought of as insurance. The time value of the option protects the holder of the option against adverse price movements in the underlying security until the option expires.

$$P_{t,t+k} = \max\begin{pmatrix} \left(p * P_u^{t+k,t+k} + (1-p) * P_d^{t+k,t+k}\right)\exp(-r * \delta t), \\ \max\left(F_{t,t+1} - X, 0\right)\exp(-r * \delta t) \end{pmatrix}, \tag{5-12}$$

where p is the previously described probability of an up movement, $P_u^{t+k,t+k}$ is the value of the put option in the next period if the underlying futures increase, $P_d^{t+k,t+k}$ is the value of the put option in the next period if the underlying futures decrease, X equals the strike price, r equals the risk free rate and δt equals represents the time that has passed from t to $t+k$ as a percentage of a year.

Consider the example is presented in figure 5-2 which considers an American put option with a strike price of $12.00 on a futures contract. The binomial tree presented in figure 5-2 has 3 steps past the current period; the third step values the option at expiration. During the current period (node time=0.0000) the price on the underlying futures is reported to equal $12.01. Each step evaluates the price of the American put option at a different point in time under different futures price scenarios. At node time 1.0000 the American put option expires and the time value of the option equals zero. At the terminal node the value of an American option equals the value of the European option because the option has expired and its value equals the intrinsic value given in equation (5-11). If the option being valued were a European style option the premium of the option at node time=0.0000 would simply be the present value of the expected intrinsic valuation at the terminal node.

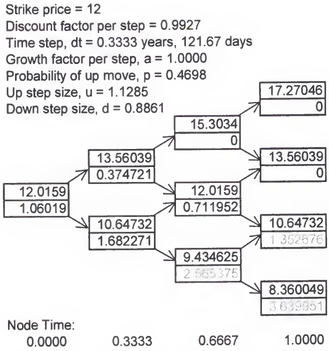

Figure 5-2. Example of futures and American options prices calculated for a three-step binomial tree.

For this example we assume the risk free rate to equal 2.19%. Consider node time=0.6667. If the underlying futures price equals 15.30 then the price of the American option equals the maximum of the present value of the expected value and the present value of early exercise. The present value of early exercise equals zero because the futures price at time=0.6667 equals 15.30 which is greater than the exercise price of 12.00. The present value of the expected value is also zero because at node 1.0 when the futures price equals 17.27 and 13.56 the value of the put option equals 0.00 and 0.00 respectively. Therefore at time node 0.6667 when the price equals 15.30 the value of the American put option equals zero. At time node 0.6667 when the value of the underlying futures equals 12.01 the value of the put option equals 0.71, which equals the discounted expected value of the option

$$0.71 = (.4698 * 0 + (1 - .4698) * 1.35) * \exp(-0.0219 * (1 - 0.6667)), \qquad (5\text{-}13)$$

because this value exceeds the present value of early exercise. At time node 0.6667 when the value of the underlying futures equals \$9.43 the value of the American put option equals \$2.56 which equals the present value of early exercise

$$2.56 = (12 - 9.43) * \exp(-0.0219(1 - 0.6667)), \qquad (5\text{-}14)$$

because value of early exercise exceeds the present value of the expected value. The value of the American put option at time period 0.3333 and 0.0000 can be found by iterating the steps above twice.

Milk Price-Supports

Price-supports complicate efforts to apply the binomial trees to dairy markets because binomial trees must be truncated to account for the price floor. Currently the price-support for milk is maintained through government purchases of butter, nonfat dry milk (NFDM) and block or barrel cheese. Purchases of supported dairy products are intended to support the price of milk at \$9.90 per hundredweight. While the milk support price has remained constant over the past few years the price of underlying supported commodities has not. The Secretary of Agriculture is charged with maintaining an effective support price of \$9.90 per hundredweight for producers. Twice a year the Secretary reviews the prices of the underlying supported manufactured dairy products. The Secretary will change the relationship of support prices for manufactured dairy products, maintaining the mandated price floor if it is determined that public expenditures on the dairy price-support program can be reduced. This adjustment is called the butter powder tilt.[6] The \$9.90 per hundredweight support price is based on milk with 3.67

[6] Two such tilts have been made since January 2001. On January 16, 2001 the Secretary dropped the support price of NFDM from \$1.00 to \$.90 per pound while simultaneously increasing the butter support

percent butterfat. This translates into roughly \$9.80 per hundredweight on a 3.5% butterfat basis. Adjustments from 3.67% to 3.5% butterfat can be made using the butterfat differential. The price adjustment from 3.67% butterfat to 3.5% butterfat milk can be calculated by subtracting 0.17 times the price of butterfat and adding 0.17 times the price of skim.[7] The support price is maintained through government purchases of storable commodities such as nonfat dry milk, butter and cheese. There is however no mandate for plants to sell to the government; therefore, in real terms, the price producers receive for delivered milk can drop below \$9.80 hundredweight. Currently plants are reluctant to sell to the government at support prices when opportunities exist in the private sector. Plants put a premium on selling milk to the private sector because the government requires special packaging and grading. Packaging and grading fees increase the cost of selling manufactured dairy products to the government. Figure 5-3 illustrates that this has resulted in class prices that are below support prices.[8] For this study it is assumed that the price-support is equal to the minimum class III price since January 2000.

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price from \$.65 to \$.85 per pound. These adjustments maintained a \$9.90 milk price-support. The government continued to accumulate NFDM well above its ability to use the product. From October 2000 to May 2001 the Commodity Credit Corporation purchased 330 million pounds of NFDM. This represented over 40% of the NFDM produced during that time period. On June 1, 2001 the Secretary revaluated the support price for butter and NFDM. The NFDM price was decreased to \$.80 per pound and the butter price was increased to \$1.05 per pound. These adjustments yielded a support price of \$9.90 per hundredweight as mandated.

[7] Price on the Chicago Mercantile Exchange are quoted using a 3.5% butterfat.

[8] Since order reform FMMO class III prices have dropped to a minimum of 8.57, 1.23 below support levels.

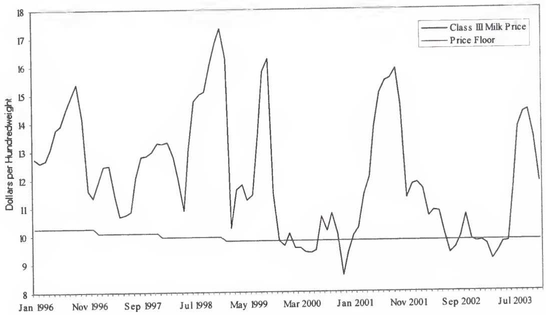

Figure 5-3. Historical FMMO class III milk prices and the price floor

The agricultural economics literature has focused on measuring the economic consequences of price-supports on consumers and producers. A typical approach outlined by Hallberg (1992) considers the welfare effects of price-supports for producers and consumers. Consider figure 5-4 where the price-support is set at P_s. The quantity demanded increases from Q_e to Q_s and the price increases from the equilibrium price, P_e ,to the support price, P_s. In the case of dairy the price-support is maintained through the removal of manufactured dairy products. In the hypothetical situation presented in figure 5-4 the cost to the government to remove the dairy product equals area *BCFG*. Producer surplus increases by areas ABED and BEC while consumer surplus is decreased by area ABED.

The problem with this kind of analysis is that it appears that producers receive no benefit when the price-support is set less than or equal to the prevailing equilibrium price P_e. In actuality even when the price floor is set below the equilibrium producers benefit

because they have an explicit guarantee that prices will not drop below the support price. The value of this guarantee can be found by finding the value of a European put option with a strike price equal to the price floor.[9]

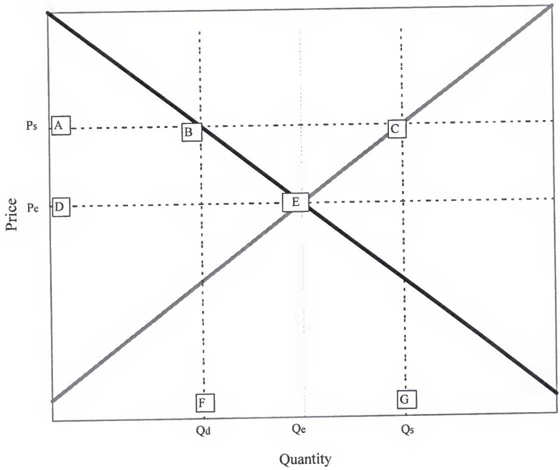

Figure 5-4. Welfare effects of a price-support with no supply control

Put Call Parity

The theoretical relationship between the price of a European put and the price of a European call with the same strike price is called put-call parity. The put-call parity relationship for options on futures contracts can be illustrated by considering two

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[9] The price floor is equivalent to a European put option and not an American put option because producers do not have the option to make early delivery. The price floor that is used is 8.57, the historical minimum of class III prices sense order reform.

portfolios. The first portfolio consists of a long European put, long futures and a cash sum equal to the present value of the futures contract. The second portfolio consists of a long European call and a cash sum equal to the present value of the strike price on the call. Mathematically the parity of portfolio one and two can be represented as (Hull 2000)

$$F_{t,t+k}e^{-r(\delta t)} + P^{E}_{t,t+k} = C^{E}_{t,t+k} + Xe^{-r(\delta t)}, \tag{5-15}$$

where $F_{t,t+k}$ is the price of a futures contract with an expiration at time period $t+k$ evaluated at time period t, $e^{-r(\delta t)}$ is a discounting factor where r is the risk free rate, $P^{E}_{t,t+k}$ is the value of a European put option on $F_{t,t+k}$ at time period t with an expiration at time $t+k$, $C^{E}_{t,t+k}$ is the value of a European call option on the same futures contract at time period t and X is the strike price for both $C^{E}_{t,t+k}$ and $P^{E}_{t,t+k}$. This implies that at expiration the payoff from a portfolio of a long futures and long put equals the payoff from a long call.

Observable and Unobservable Components of Class III Milk Futures

The payoffs from futures contracts traded on class III milk behave in way that is very similar to a call option. The price floor for milk acts as a European put option for processors of manufactured dairy products. Whenever the wholesale prices of commodities such as butter, cheese and nonfat dry milk powder fall below support prices the manufacturers of these commodities "exercise their put option" and sell their products to the Commodity Credit Corporation (CCC) at support prices. The traders of milk futures and options recognize that a floor exists and trade accordingly. Therefore, traders of long futures are actually trading a portfolio of a long futures contract and a long put

option. The sum of the gains from each of these commodities is observable as the price reported by the Chicago Mercantile Exchange on Class III Milk Futures. The components of this portfolio: a long futures contract and a long European put option are unobservable because they are not traded. For the purposes of this paper the observable actual price reported by the exchange on class III futures shall be denoted as $F^{III}_{t-k,T}$ and the unobservable theoretical components consisting of a unobservable futures contract and unobservable put option will be denoted as $F^{IIIU}_{t-k,T}$ and $P^{IIIU}_{t-k,T}$ respectively, where k is the approximate duration of the hedge.

Callibration of Unobserved Components

This is a valuable observation because it allows for the use of binomial trees to price American options on class III milk. The binomial down step, D, is a constant. One of the underlying assumptions of the binomial tree model is that prices have a lognormal distribution that is truncated at zero. This assumption is violated for dairy because price-supports prevent prices from ever reaching this point. One solution to this problem is to construct a binomial tree based on the unobservable futures $F^{U}_{t-k,T}$. The tree that is constructed is then used to price a European put option with a strike price that equals $8.57.

Start by setting the unobserved futures price equal to $12.06, which was the price on November 13, 2000 for a November 2004 class III milk futures contract, and the strike price on the unobserved European put equal to $8.57, the lowest class III price observed. Assume that the implied volatility of the observed futures equals the volatility of the

unobserved futures because both contracts are based on the same market.[10] Construct a synthetic portfolio that consists of one unobserved long futures and one unobserved put. Value the unobserved put using a binomial tree. The value of the synthetic portfolio equals the value of the unobserved futures contract plus the value of the unobserved European put option valued by the binomial tree. Iterate using different underlying unobserved futures prices until the synthetic portfolio equals the value of the observed futures price. Model calibration can be made using a mathematical program but can be accomplished easier using an iterative approach.

The error of the model is defined as the sum of the absolute difference between the predicted American option price and the actual American option price and the absolute difference between the sum of the unobserved futures and European options and the observed futures. Mathematically this is formulated as

$$\begin{aligned}
&Minimize \;\; Error = \left|F_{t-k,T}^{III} - F_{t-k.T}^{IIIU} - P_{t-k,T}^{IIIU}\right| + \left|P_{t-k,T}^{III} - P_{t-k,T}^{IIIB}\right| \\
&By\ Changing: \\
&\qquad\quad \sigma_T \\
&\qquad\quad F_{t-k,T}^{IIIU} \\
&Subject\ to: \sigma \geq 0 \\
&\qquad\quad F_{t-k,T}^{IIIU} \geq 8.57
\end{aligned} \qquad , \qquad (5\text{-}16)$$

where $P_{t-k,T}^{III}$ equals the actual American put option premium, $P_{t-k,T}^{IIIB}$ equals the American put option premium predicted by the modified binomial trees and σ_T equals the implied volatility of the model. The decision variables are constrained. The volatility is constrained to be positive and the value of the unobserved futures is constrained to be

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[10] Imply volatility using a put option with a strike price that is just outside of the money. Option premiums become less sensitive to volatility as the current futures price and strike price diverge. Therefore in order to get an accurate representation of volatility select an option with a strike price that is just out of the money was selected to imply volatility.

greater than the price floor. The unobserved futures are constrained to ensure a global minimum.

Model calibration can be made using a mathematical program but can be accomplished easier using an iterative approach. The math program presented in (5-16) is solved using a grid approach. A grid is constructed for likely values of the unobserved futures and volatility. The error is then calculated for all points on the grid. The point on the grid that gives the minimum value is the point that is used to calibrate model. For this study the value of the objective function in (5-16) had to be less than or equal to

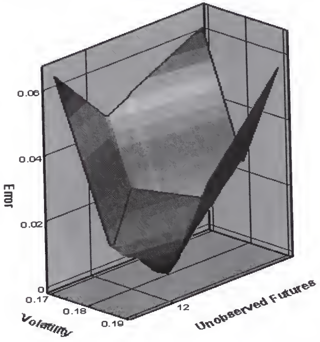

Figure 5-5. Three dimension graph representing calibration error for various volatility and unobserved futures combinations.

0.005 in order to be used. This constraint ensured that the combined error of the binomial prediction of the American put option premium and the binomial prediction of observed futures did not exceed half of a penny. It is assumed that this level of accuracy would be more than adequate. An example of this procedure is illustrated for k=6 in figure 5-5. If this level of precision could not be met then the process was continued using a finer grid.

The final premium on the unobserved put equals the value of the price floor to dairy producers. This approach complements figure 5-4 by clarifying the value of the explicit guarantee given by the government to producers.

Implementation

For storable commodities the theoretical basis converges over time. This is not necessarily the case with non-storable commodities such as class III milk. Milk produced this period cannot be stored and sold in the following period in order to "make delivery" on a futures contract obligation. The result is that the predictable basis patterns that exist

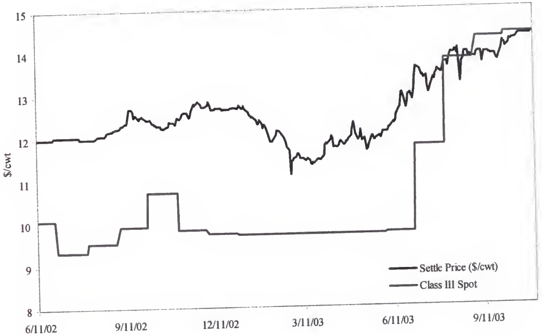

Figure 5-6. Basis pattern for October 2003 futures and class III milk

for storable commodities such as wheat and corn do not exist for non-storable commodities such as milk. A typical basis pattern for milk over time is represented in figure 5-6 which plots the basis for October 2003 over time.

Despite the erratic basis behavior over time there is one point in time when the basis does equal zero and that convergence is found at expiration. The classified pricing system ensures that all processors pay the minimum price for milk utilized in the class III market. Because the minimum price paid by producers is equal across all markets spatial impacts on the closing basis are negligible. Furthermore the contract specifications for class III milk specify that contracts cash settle to the minimum cash price released by the USDA. Cash settlement effectively eliminates costs associated with "delivery". These two features ensure that the closing basis always converges to zero at contract expiration for class III milk futures. Therefore because at expiration the closing futures price $F_{T,T}^{III}$ and cash price C_T^{III} must always be equal the implied volatility of $F_{T,T}^{III}$ equals the implied volatility of C_T^{III}. Given that the futures are unbiased predictors of closing futures prices, future cash prices, and future price volatility for both futures and cash markets, the binomial model that is constructed can infer a lot about the characteristics of future prices.

Consider figure 5-7 which is based on an observable futures contract. Prices on the unobservable futures contract are assumed to have the same volatility as the observed futures contract but are defined from 0 to positive infinity. Figure 5-7 illustrates a five step binomial tree where the predicted prices at time node 12/02/04 range from \$7.43 to \$19.44. The lower bound of this abbreviated tree drops below the assumed support price of \$8.57. If the unobserved futures price drops to \$7.43 at time node 12/02/04 then the intrinsic value of the European put option will have a value of \$1.14. The sum of the value the unobserved European put and unobserved long futures equals the price floor of \$8.57. The model is calibrated such that the sum of the unobserved futures and the

unobserved European put equal $12.06 for node time 11/13/03. This is the value of November 2004 futures valued on November 13, 2003. An American put option with a strike price equal to $12.00 is valued using valuation formula presented in equation (5-12). In reality a five step model does not sufficiently define the spectrum of prices that producers are exposed too, therefore for this study a 97 step binomial tree was used.

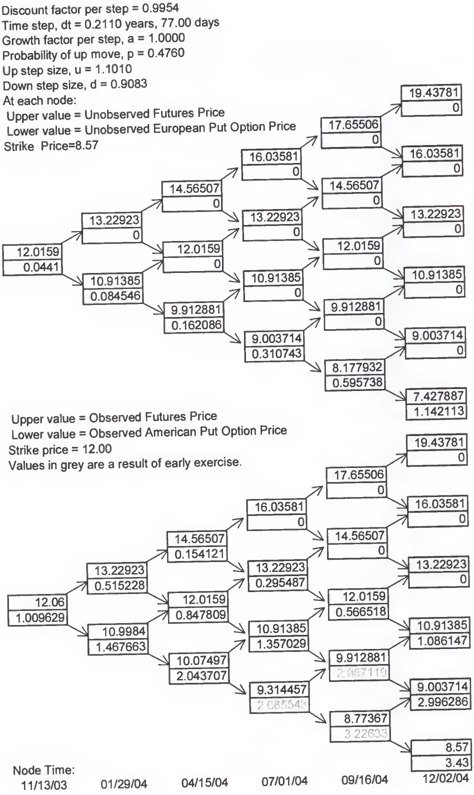

Figure 5-7. Example of an abbreviated binomial tree constructed for unobserved futures, options and observed American options on observed futures

CHAPTER 6
PRECURSORY, IMPLICIT, AND EXPLICIT MODEL RESULTS

Introduction

Chapter 6 will highlight a number of different results. Results fall into one of three categories. The first section of chapter six will present results that were derived for the sole purpose of model development. Precursory results include historical utilization, production and seasonality trends in addition to the effectiveness of various hedging strategies. The second section reports results implied by the model. This section reports the past, present and future implied price volatilities for class III milk as well as the past, present and future valuations of the milk support program. The third section of chapter 6 will report the producer's ability to hedge under four different scenarios. Each of the following scenarios is evaluated under three different capital structures (0, 25 and 50% debt), using various futures and options hedging scenarios.

- No coverage from the Milk Income Loss Contract Program and production certainty, (NMLCNPR).
- No coverage from the Milk Income Loss Contract Program and production risk, (NMLCPR).
- Full coverage under the Milk Income Loss Contract Program and production certainty, (MLCNPR).
- Full coverage under the Milk Income Loss Contract Program and production risk, (MLCPR).

Precursory Results Derived for Model Development

Utilization

In order to predict milk revenue the mailbox price needed to be estimated. The mailbox price was estimated as the sum of the blend price and the over order premium minus a correction for marketing fees. The blend price equals the average of milk prices

in each class weighted by utilization. This section summarizes the seasonal and long term trends found in milk utilization. These results were used to forecast future milk utilization.

Over 99% of the variation in milk utilization can be accounted for by seasonal variation and variation due to time. The multivariate linear regression was estimated using restricted least squares. Several restrictions were imposed on the model. The additional information in the restrictions reduced the variation in the parameter estimates, increasing their overall significance.

Class I utilization has decreased in recently years while class II, III, and IV utilization estimates have increased. Trends in the class I, II, and class III market are significant at the 99, 95 and 95% confidence levels respectively. Marked seasonal variation also exists in the intercepts for class utilization. When the model is run restricting the quarterly intercept terms for each class utilization to equality the error sum of squares increases from 0.06197 to 0.08826. The resulting F statistic ($F(12,160) = 5.657$) was used to test whether there exists seasonal differences in the class intercepts. The F test indicates that the seasonal variation is significant at the 99% confidence interval. For a more detailed discussion on the regression used to predict utilization please reference appendix A.

Milk Production Per Cow

In order to predict milk revenue, milk production per cow needed to be estimated. This section summarizes the seasonal and long term trends found in milk production and is based on the econometric model in appendix B. These results were used to forecast future milk revenue.

The variation in agricultural production is predominantly determined by variation in trend and seasonality (Harwood 1999). The first type of variation faced by Florida milk producers is the gradual increase in milk production over time. This gradual increase was modeled against the natural log of the number of months since January 1987. The seasonal variation was modeled against the sine and cosine of the number of months since January 1987[1]. The resulting error terms were then grouped by month. This grouping allowed for the specification of production risk.

Production risk for this model was based on the models ability to predict future milk production per cow. The specific level of production risk was estimated using the conditional value at risk. The results are presented in figure 6-1 illustrate two things. First of all seasonality is a function of weather patterns. Secondly, figure 6-1 illustrates that prediction of milk production per cow is much better during some months than during others. There are many uncertainties that face Florida dairy producers. Hot, humid summer weather is not one of them. Milk production per cow is seasonally low during the months of August, September and October. While October is typically a low month for milk production it is very difficulty to characterize because it marks the transition from Summer to Fall. This period is notable not only because of the low expected production but also because of the high level of production risk. Production risk is important because of its implications for hedgers. In general hedge effectiveness is higher when production risk is low. The conditional value at risk (10%) risk measure was applied to milk production per cow data. Figure 6-1 illustrates the relationship

[1] Production data extending back to January 1987 was used because it was readily available and because it was assumed that that 17 years of production data would be enough to give a good approximation of production risk.

between the expected detrended milk production per cow, the conditional value at risk (or maybe it should be called the conditional production at risk), and the percentage difference between the two. On a percentage basis the conditional value at risk relative to the expected detrended milk production is greatest for October indicating that production risk is also greatest during this month.

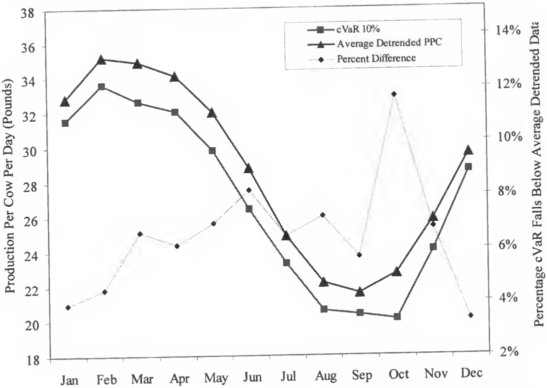

Figure 6-1, The average detrended milk production per cow, conditional value at risk (10%) and percent difference for each month.

Hedging Strategy and Closing Basis

The producer's ability to lock in a price is determined solely by the closing basis. The closing basis can be broken down into two components the mover effect and the acceleration effect. The mover effect only works in the producer's advantage while the acceleration effect introduces the possibility of a negative closing basis which can reduce the producer's effective price. Producers' should seek a strategy that maximizes the

explanatory power of the mover on closing basis. Producers control the date that they offset their short positions and they also control the contract that is used to hedge. This section will detail the hedging effectiveness of six different hedging strategies.

Assume that a producer attempts to hedge class I milk production for a month ending at *T* using futures at time period *T*-6. The price that the producer attempts to lock in equals $F^{III}_{T-6,T}$, where *T* also equals the date that the futures contract expires. The effective price for that producer then equals the class I mover C^{I}_{v}, where *v* equals the date that the class I mover is released, plus the gain on futures. The gain on futures equals $F^{III}_{T-6,T}$ - $F^{III}_{v,T}$. Since the producer already knows $F^{III}_{T-6,T}$ the producers effective price is determined solely by the difference between the stochastic C^{I}_{v} and $F^{III}_{v,T}$. This difference can also be referred to as the closing basis. Given the nature of the pricing relationship between the class I and class III market it is expected that the closing basis will usually be positive. This result is expected because the class I mover is based on the maximum of the class III and class IV advanced prices. However, when the historical data is considered it is found that this is not the case. Consider figure 4-3 which plots the effective price versus the class III enter price for a class I hedger who attempts to lock in the class I mover six months prior to the date it is released. Figure 4-3 is drawn based on the assumption that the producer is hedging the mover using lagged class III futures contract and offsetting the position on the day that the class I mover is announced.

In general the effective class I mover is greater than the class III enter price. However on March 2002 and on November 2002 the effective price was less than the class III enter price. This result occurred because C^{I}_{v} was less than $F^{III}_{v,T}$. This is

plausible because of the acceleration effect. The acceleration effect results because C_v^I and $F_{v,T}^{III}$ are based on different survey periods.

The mover effect was calculated as the maximum of the difference between the class IV advanced price and the class III advanced price and zero. The acceleration effects of the following six different strategies were evaluated.

- Current futures, offset on the day the first NASS survey, used to calculate the class I mover, is released
- Current futures, offset on the day the second NASS survey, used to calculated the class I mover, is released
- Current futures, offset on the day the class I mover is released.
- Lagged futures, offset on the day the first NASS survey, used to calculated the class I mover, is released
- Lagged futures, offset on the day the second NASS survey, used to calculate the class I mover, is released and
- Lagged futures, offset on the day the class I mover is released

Each strategy was evaluated using the conditional value at risk.

The closing basis is plotted against the mover effect using strategy three in figure 4-4. For this study the difference between the closing basis and the mover effect is called the acceleration effect. When different strategies are used the closing basis and therefore the acceleration effect changes. The acceleration effect for the current (lagged) futures strategies are presented in figure 6-2 (6-3).

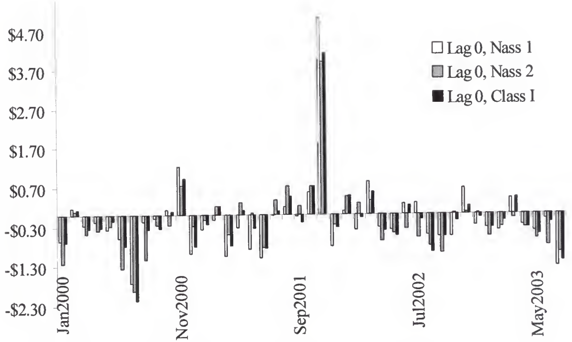

Figure 6-2. The acceleration effect for strategies using current futures

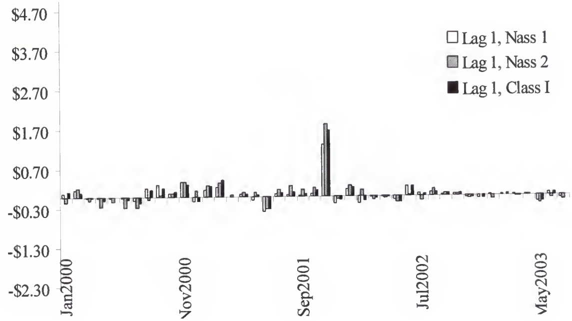

Figure 6-3. The acceleration effect for strategies using lagged futures

Applying the conditional value at risk (10%) to the distribution of the acceleration effect for each of the six strategies it is apparent that the strategy that minimizes negative basis risk from the acceleration effect is the strategy where milk production is hedged using a lagged futures contract and the contract is offset on the day that class I prices are released. The conditional value at risk for each of the six respective strategies equals -1.508, -1.603 , –1.452 -0.2476, -0.3204, and -0.225 respectively. Since the conditional value at risk is minimized for strategy three that is the strategy that was used for this study. Strategy three uses the lagged futures offset on the day class I prices are released. When strategy three is used nearly 95% of the closing basis is explained by the mover effect.

Implicit Model Results

Milk Price-Support

The stochastic distribution of future class III milk prices was derived using the binomial tree. However as noted in chapter 5 the milk price floor violates Cox and Rubenstein's assumption that the underlying asset's future price distribution is lognormally distributed and not truncated. The price floor effectively truncates the distribution introducing upward bias. The correction for this described in chapter 5 involved modeling observed prices as the portfolio of an unobserved futures and an unobserved European put contract. The premiums and implied volatilities that are implied by calibrating the binomial trees, yield interesting results that have special economic significance.

In order to determine the value of the price floor to producers, a number of steps had to take place first to generate the appropriate data sets. First of all, the futures and options class III prices from October 1, 1999 to January 23, 2004 for contracts expiring

between April 2000 and June 2004 were retrieved. Secondly, put option premiums were retrieved on the same day for a put option at or just in the money. Thirdly, thirteen-week t-bill auction data was used as a proxy for the risk free rate. The risk free rate was approximated for a given month by looking up the investment yield on 13 week t-bills issued that month. Prices on futures contracts were looked up about 180 days prior to expiration. These data were used in conjunction with the binomial tree described in chapter 5 in order to imply the volatility for each of the months. The iterative procedure used to determine the unobserved futures prices and unobserved put option price was repeated for all the months from April 2000 to June 2004.

Past, Present, and Future Implied Price Volatilities

Figure 6-4 shows the implied volatilities using the procedure defined above. The implied volatility of class III dairy prices ranges from 11% to 38%. There does not appear to be an obvious seasonality or trend to the implied volatilities. The background of figure 6-4 shows a contour, which represents the value of the price floor 180 days prior to the expiration date of the class III milk contract. Holding volatility constant, the change in the value of the contour for a given change in the unobserved futures yields delta (Hull 2002). As the value of observed futures drops delta increases in magnitude. That is to say the producers valuation of the price floor is more sensitive when the value of observed futures is low (closer to the price floor) than when it is high. The sensitivity of the price of this portfolio to changes in volatility is frequently referred to as Vega (Hull 2002). Vega equals the change in the value of the portfolio for a given change in the level of volatility and can be measured for a fixed observed futures level. This relationship is can be seen in figure 6-4. The distribution of the producer's price floor valuation can be found in figure 6-5.

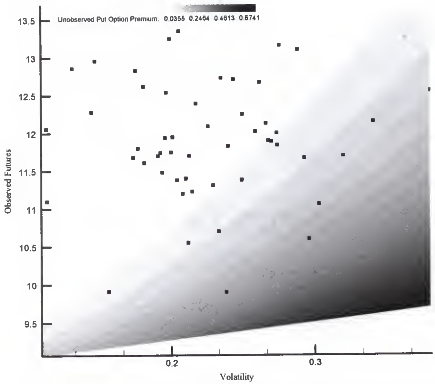

Figure 6-4. The implied volatility, observed futures and unobserved put options ($8.57 Strike) values 180 days prior to expiration for April 2000 to June 2004 futures.

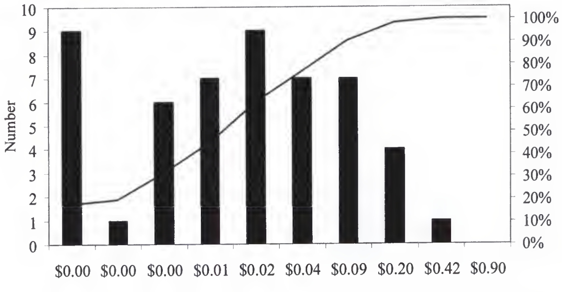

Figure 6-5. The distribution of the producer's valuation of the price floor ($8.57) 180 days prior to expiration (based on April 2000 To June 2004 class III futures).

Explicit Model Results

For this study a sample size of 1020 different net income scenarios were used. The minimum risk futures and options hedge ratios were calculated under each of four different MILC and production risk combinations. Due to the small sample size 100 different samples each consisting of 1020 net income scenarios were drawn. This bootstrapping exercise attempted to demonstrate the variation in the minimum risk futures and options hedge ratio for each of four scenarios (listed at the beginning of this chapter) that resulted from sampling error. The minimum risk futures (options) hedge ratios calculated from each of the 100 bootstrap samples are presented in figure 6-6 (6-7).

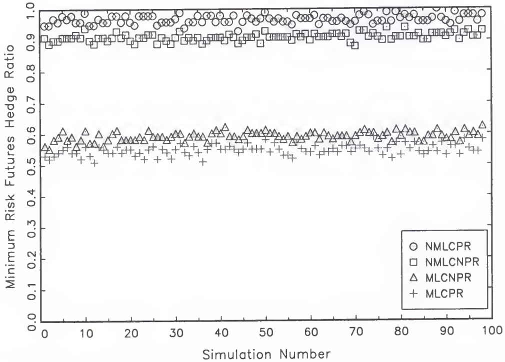

Figure 6-6. Calculated minimum risk hedge ratios for 50 samples under four different scenarios.

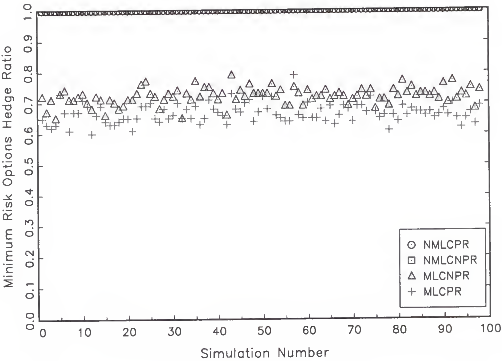

Figure 6-7. Calculated minimum risk options hedge ratios for 50 samples under four different scenarios.

The average (calculated from the bootstrap samples in figure 6-6) minimum risk futures and options hedge ratios as well as the corresponding conditional value at risks for each of the four scenarios are listed in table 6-1. The minimum risk futures ratio equals 0.967 and yields a conditional value at risk of $22,166.13 for a producer that is not covered by MILC and has production certainty. The minimum risk options hedge ratio for bonafide hedgers equals one. Although if producers are not constrained to a bonafide hedge positions their risk could be further reduced by hedging more than 100% of their expected production. This apparent anomaly is an idiosyncrasy of the conditional value at risk. This result indicates that producers can reduce their risk by over hedging thus over compensating for losses. For this study it is assumed however that producers will not hedge more than their expected production and are therefore constrained to bonafide hedging positions. Producers not covered under the MILC program facing production

risk have a minimum risk hedge ratio of 0.911 and a cVaR of $20896.93 both of which are less than the minimum futures hedge ratio under production certainty. Producers who are enrolled in the MILC program hedging under production certainty have less risk but have less flexibility to hedge remaining risk. The minimum risk futures ratio equals 0.587 and the minimum options hedge ratio equals 0.713 and yield conditional value at risks of $33,889.15 and $22,332.73 respectively. When production risk is included for the producer covered under MILC the minimum risk futures and options hedge ratios drops to 0.543 and 0.657 yielding a conditional value at risk of $32,439.92 and $27545.74 respectively.

Table 6-1. Average minimum risk futures and options hedge ratios and corresponding conditional value at risk (calculated from bootstrap sample)

Scenario	Figure	Minimum Risk Futures Hedge Ratio	Futures cVaR	Minimum Risk Options Hedge Ratio	Options cVaR
NMLCNPR	Figure 6.8	0.967	$22166.13	1.000	$11496.77
NMLCPR	Figure 6.9	0.911	$20896.93	1.000	$11333.98
MLCNPR	Figure 6.10	0.587	$33889.15	0.713	$28332.73
MLCPR	Figure 6.11	0.543	$32439.92	0.657	$27545.74

The following sections will further elaborate on these four different scenarios. Comparisons will also be made for different capital structures. For each section a representative sample was drawn. The representative sample drawn closely approximated the mean minimum risk hedge ratio in figure 6-6. Figures 6-8 and 6-9 show the risk surface for a producer not covered by MILC under production certainty and production risk respectively. Figures 6-10 and 6-11 show the risk surface for a producer receiving full coverage from MILC under production certainty and production risk respectively. The contours that are formed in each of the figures represent iso conditional value at risk levels. Risk levels decrease contour level increases in

magnitude. The minimum risk futures hedge ratio be found by finding the greatest contour on the futures axis. This contour encircles the minimum risk futures hedge ratio. Similarly the minimum risk options hedge ratio can is achieved by the greatest contour that intersects the options axis

Producers are always able to achieve a lower risk level (higher conditional value at risk level) using futures than options. This is the result of expensive option premiums. Furthermore the minimum risk options ratio always exceeds the minimum risk futures ratio. For this study it is assumed that producers are constrained to assume a bona fide hedge position (hedge must be less than or equal to 1). This constraint can be represented graphically as a 45 degree line with a slope of –1 and intercept of 1 which can be drawn on the diagonal of figures 6-8, 6-9, 6-10, and 6-11 respectively.

The negative slope of the iso conditional value at risk contours illustrates the trade off between options and futures necessary to achieve a given risk level. Some producers prefer options instead of futures because options allow the producer to still reap the benefits of high prices even when their respective commodity is hedged. Furthermore, put options are not subject to margin calls. On the other hand premiums for dairy options are notoriously expensive.

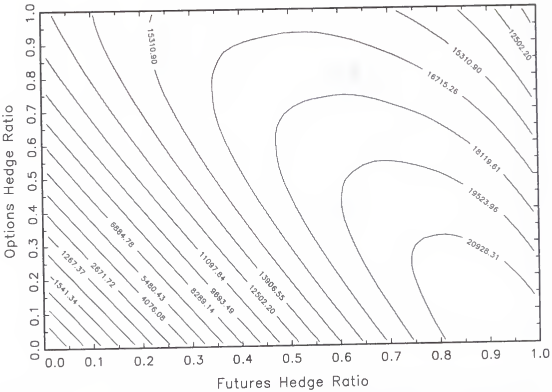

Figure 6-8. Iso-cVaR contours for hedging under production certainty with no MILC coverage

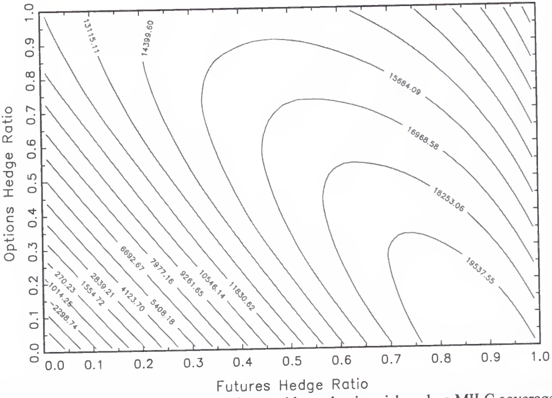

Figure 6-9. Iso-cVaR contours for hedging with production risk and no MILC coverage

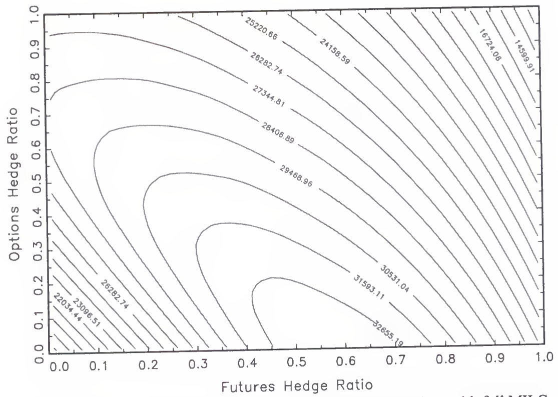

Figure 6-10. Iso-cVaR contours for hedging with production certainty with full MILC coverage

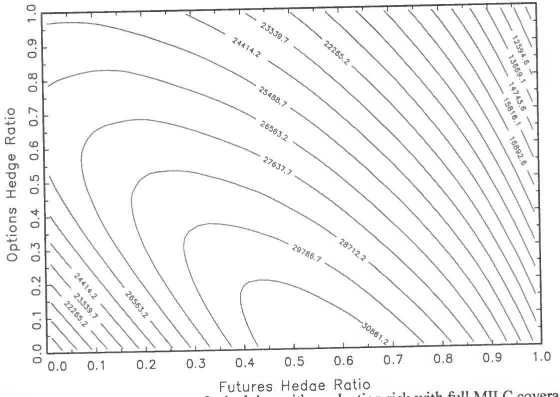

Figure 6-11. Iso-cVaR contours for hedging with production risk with full MILC coverage

The risk balancing hypothesis states that producers will attempt to use financial risk to compensate for increased or decreased business risk. This section will determine whether the converse is possible. Is it possible to use futures or options to reduce business risk in order to react to increased financial risk? Figure 6-12 shows the different hedging positions under production risk and production certainty for producers not enrolled in MILC that yield a conditional value at risk of 0. The three capital structures that are considered are 0 ,25, and 50% debt.

The addition of debt only shifts the respective iso-cVaR curves down and therefore it does not impact the minimum risk futures or options hedge ratio. This is a result of the translation invariance property of the conditional value at risk. Producers with production risk need to hedge a greater percentage of their expected production than producers producing under production certainty. Similarly producer with debt always need to hedge a greater percentage of their expected production than producers without debt need to hedge in order to achieve a given risk level.

Figure 6-13 shows the various hedge position necessary in order to achieve a conditional value at risk of $20,000 for producers under production certainty and production and enrolled in MILC with 0 , 25 and 50% debt. Noteworthy is that no hedging position allowed for the producers under production risk and 50% debt to reach the risk levels of the other scenarios. Therefore it is noted that there are limits to the producer's ability to balance risk.

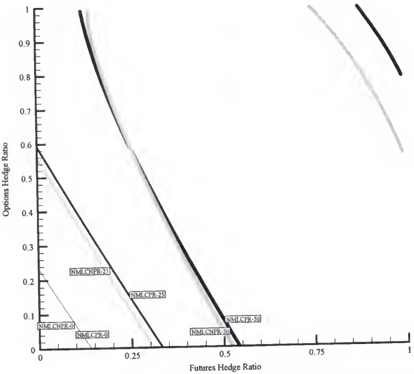

Figure 6-12. Hedge portfolios yielding a \$0 cVaR (10%) for producers not covered by MILC, facing production certainty or production risk with 0,25, and 50% debt

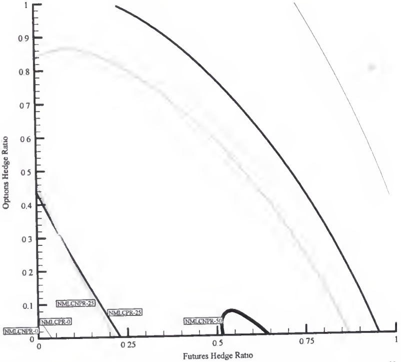

Figure 6-13. Hedge portfolios yielding a \$20,000 cVaR (10%) for producers with full MILC coverage, facing production certainty or production risk with 0,25, and 50% debt

CHAPTER 7
SUMMARY, CONCLUSIONS, IMPLICATIONS, AND DIRECTION FOR FUTURE RISK MITIGATING RESEARCH IN THE FLORIDA DAIRY INDUSTRY

Introduction

The final chapter of this dissertation will be broken down into four sections. Each of the first three sections will attempt to summarize, draw conclusions and postulate implications about the research objectives stated in chapter 1. The first section will highlight results that were used to estimate milk utilization and milk production parameters in the model. The second section will highlight assumptions made in the model regarding hedging strategy and the milk price floor. The third section will provide insight into the empirical results that were yielded from the model. Finally the fourth section will outline one possible direction that future risk mitigating research on the Florida dairy industry could take.

Summary, Conclusions, and Implications Regarding Model Parameters

Milk Utilization

The parameter estimates for milk utilization presented in the previous chapter can by summarized in two points. First of all, a decrease in class I milk utilization has been occurring over time. The decrease in class I utilization is offset by increased utilization in each of the other three milk classes. The second point is that seasonality exists in milk utilization. During the second quarter less milk is utilized in the class I market than during other months. Similarly, during the third quarter, a higher percentage of milk is utilized in the class I market.

Several conclusions can be drawn from these results. First of all, during months when there is a high class I utilization producers need to hedge a lower percentage of their expected production in order to reach a certain threshhold risk level. This is because producers are expected to earn a higher blend price during months with high class I utilization. Secondly overtime as class III utilization increases producers will face less basis risk (acceleration effect) and therefore producers will be able to set a more effective hedge (more effectively lock in a milk price).

The implications that government can draw from this is that the risk mitigation program designed for Florida producers should consider the impact of season.

Milk Production

Milk production per cow in Florida can be summarized by the first and second moment of its distribution. The seasonality of the average milk production per cow in Florida confirmed by this research is well documented. While many articles have focused on the seasonality of the first moment of milk production per cow much less focus has been concentrated on the second moment.

The second moment is frequently used as a risk measure.[1] Chapter three focused on the theoretical superiority of the conditional value at risk as a risk measure. In order to be consistent the conditional value at risk for various distributions for various production months is reported instead of the variance. The academic literature tells us that in cases where the distributions are not skewed the conditional value at risk is just about as effective as the variance when it comes to evaluating risk. However, in reality there is no reason to suspect that milk production per cow has a skewed distribution.

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[1] The second moment equals the variance when the expected value equals zero.

Furthermore, if the underlying distribution is normal (as the central limit theorem suggests is very likely) then the conditional value a risk and variance are but multiples of each other.

To summarize the results indicate that variation does exists in the conditional value at risk estimates of milk production per cow from month to month. The regression used to estimate milk production per cow indicates that it is most difficult to predict milk production during October and too lesser degrees May and June.

The high production risk during October, May, and June leads to the conclusion that producers are vulnerable to production risk because these months are transition months. In Florida transition months are sometimes hot enough to stifle milk production, but not always. Therefore, because of ambiguous weather during these months, production risk is higher for producers.

The varying degrees facing producers has hedging implications. The academic literature has shown that production risk increases the producer's exposure to risk, furthermore production risk reduces the minimum risk hedge ratio. The finding that milk production risk varies from month to month means that a "one size fits all" risk management strategy might not be appropriate.

Summary, Conclusions, and Implications Regarding Model Assumptions

Hedging Strategy

The class I producer's ability to lock in a minimum price is a function of the convergence of the class I mover and the class III futures price. To summarize, this research has found that when the hedge position is set using a lagged futures position and is lifted on the day the class I mover is released, the probability of convergence (or divergence in the producer's favor) is maximized. Furthermore convergence is more

likely in certain months than others. This result occurs because during some months, five weeks of survey data are used to calculate the class III price while during other months only four weeks of survey data are used. Months in which only four weeks of survey data are used to calculate the FMMO class III prices are more likely to converge. This result occurs because the two NASS surveys used to calculate the class I price carry more relative weight in months where only four surveys are used. These results lead to the conclusion that the sensitivity of the hedge to these factors is primarily a function of the perishable nature of milk. Non storable commodities have erratic basis patterns, which make the effectiveness of the hedge very sensitive to the date the producer offsets his or her hedge position.

Even when setting a hedge using lagged futures and offsetting on the date class I prices are released, convergence is not assured for three reasons. First of all, the class I mover is based on the maximum of the class III and class IV mover (mover effect). Secondly, the class I mover is based on a shorter survey period than the class III price (acceleration effect) and third, the class I mover is released on a different day than the class III price (acceleration effect). The closing basis results from divergent prices on the day that class I prices are released. The closing basis is composed of a mover effect, which only benefits producers, and an acceleration effect which can work to the producers disadvantage. The recommended strategy of using lagged futures and offsetting the position on the day the class I mover is released was derived by minimizing the detrimental conditional value at risk of the acceleration effect.

Empirically the recommended strategy minimized the producers exposure to the acceleration effect. This research concludes that the sensitivity of closing basis to this

strategy is a function of the bulkiness and perishability of fluid milk. Given the non storable nature of milk NASS surveys can vary significantly from month to month yielding an erratic basis. This variation yields an acceleration effect.

The implication is that policies might be considered in order to reduce the acceleration effect would be to include more than two weeks of NASS survey data in the computation of class I prices. Increasing the number of survey periods (so long as the additional survey period was used to calculate class III prices for the same month) would reduce the detrimental impact of the acceleration effect; however this policy would also compromise the advanced nature of the class III prices. Another policy initiative that would also reduce the acceleration effect would be to reduce the number of surveys used to calculate class III prices so that more consistency exists between the surveys used to calculate the class I mover and the class III prices. However this policy could be prone to manipulation because processors could time their sales to coordinate with survey months thus reducing prices for producers during those months.

Price Floor

Recently the minimum FMMO class III milk price has dipped to $8.57, well below the price floor of $9.80 per cwt (3.5% butterfat). This has forced many producers to question the integrity of the price floor particularly given the magnitude of the price floor penetration. The value of the milk price floor to producers has become uncertain.

Summarizing the methodology employed in chapter 5 to value the price floor, the values the observed class III futures prices can be modeled as the sum of an unobserved futures contract and an unobserved options contract. The strike price used for the unobserved options contract equals $8.57 or the minimum class III FMMO price since

January 2000. It was assumed that the true price floor equals $8.57 and not the federally mandated $9.80 per hundredweight.

Results based on this methodology are summarized in chapter 6. Based on this assumption it was found that the producer's valuation of the price floor six months prior to production ranged from 0.00 to 0.09 cents per hundredweight. An additional piece of information reported by the model is the implied volatility of the class III market. From April 2000 to July 2004 implied volatilities have forecasted volatilities ranging from 11% to 38%.

Based on this valuation procedure the conclusion is reached that the producer's valuation of the price floor is very sensitive to the strike price. In order to reap the full production stabilizing benefits of the price floor the government should clearly communicate the milk price floor is for delivered class III raw milk.

The recent price floor penetration by FMMO prices implies one of two things. Either the support prices for cheese, butter, and nonfat dry milk are too low or restrictions make delivery to the commodity credit corporation more costly than delivery to the private sector. In the meantime however, conservative valuation of the price floor leaves plenty of room for Florida producers to manage their risk position using class III futures and options.

Summary, Conclusion, and Implications Regarding Model Results

Milk Income Loss Contract

To summarize, this research has found that coverage under MILC is similar to the coverage that a producer would attain by purchasing a European put option (for 45% of milk production) with a strike price of $13.69 where the underlying asset equals the class I mover. This research has found that enrollment in MILC impacts the producer in two

ways. First of all, enrollment in MILC significantly reduces the producers risk thereby reducing the need to hedge. Secondly, the minimum risk hedge is decreased.

Given the sensitivity of the minimum risk hedge ratio to enrollment in MILC this research concludes that Florida's producers need to consider their coverage by MILC before setting a hedge. Given the average herd size of the Florida dairy producer, the risk minimizing hedge will vary from month to month depending on coverage under MILC. The MILC program covers the first 2,400,000 pounds of milk shipped. A Florida producer milking 1000 cows producing 50 pounds of milk a day would exhaust this coverage in a little more than a month. Equivalently the maximum coverage equals 45% of expected production but can be less during the producer's last month enrolled in MILC, depending on the amount of residual coverage. In Florida, it is likely that during some months the producer will be covered by MILC, while during other months of the year the producers will likely have no coverage. Furthermore, there will be at least one month where coverage could vary anywhere from 0 to 45% of expected production. Therefore, given the structure of MILC and the size of the average Florida dairy farm, one could conclude that utilization of futures is more complex for Florida producers than for smaller producers in other parts of the United States.

This conclusion leads to the policy implication that governments need to be sensitive to the various degrees of risk facing Florida producers. In 1996, the Freedom to Farm bill authorized the Dairy Options Pilot Program (DOPP). The DOPP encouraged producers to acknowledge their risky environment and to proactively use options to manage risk. Little guidance was given to producers telling them how much they should hedge. This is probably because it was not necessary because producers reduced their

risk so long as they did not hedge more than 100% of their expected production. For this study producers not enrolled in the MILC program had an options hedge ratios equal to 1. However, with the introduction of MILC the model results report that the minimum risk hedge ratio is less than 1. Producers who attempt to hedge and do not consider their position in MILC could easily increase their risk position. The implication for future policies is that so long as policies like MILC are employed the government should ensure that programs such as DOPP do not encourage producers to make hedges that actually increase their risk position.

Production risk

The results indicate that production risk does impact the producer's hedge ratio. Production risk increases the producers risk exposure and reduces the minimum risk hedge.

One might conclude that in reality production risk plays a much greater role. For this study aggregated production data was used to estimate production risk. Aggregated data underestimates the true production risk faced by producers. In reality the increase in risk and the decrease in the minimum risk might be more substantial.

Although production risk impacts the minimum hedge ratio in a less dramatic way than other scenarios tested the implication is clear. Highly risk adverse producers should consider the impact of production risk on the minimum risk hedge position. Risk averse producers should not value only production enhancing technologies, but also production stabilizing technologies.

Capital Structure

To summarize the results, risk balancing is possible within limits. The introduction of debt requires that the producer make interest payments. These interest payments

increase with leverage and shift the distribution of net income to the left. In order to maintain a given conditional value at risk it is necessary for producers to increase their hedge ratio in order to balance risk. This research has demonstrated that a certain degree of risk balancing is possible for producers carrying between 0 and 50% debt however, the range for which risk can be balanced shrinks substantially when producers are enrolled in MILC.

This result forces us to conclude that producers that hedge can use more leverage and still stay within reasonable risk tolerances than producers that do not hedge. Caution should be exercised; lenders need to recognize the limits of risk balancing and the interaction between the producer's ability to risk balancing and enrollment in MILC.

The implications to lenders in Production Credit and Federal Land Bank is that the conservative lending policies adopted in the 1980's need to be modified in order to allow producers, who mitigate risk using futures and/or options, better access to debt. Lenders need to have a better understanding of futures and options and the dairy industry. Limited usage of futures and options means that few lenders do have this understanding. This is unfortunate, because not only does it deprive producers of the opportunity to use more leverage but it also greatly complicates hedging. Hedging is made more complicated when the producer does not team up with a knowledgeable banker because of the margin calls that the producer needs to meet when prices increase.

Directions for Further Risk-Mitigating Research on the Florida Dairy Industry

The final section of this dissertation will compare the class III hedger with the class I hedger. This section will demonstrate the unique risk facing the class I hedger. The evolution of milk pricing over time has resulted in a complex system of formulas designed to streamline the marketing of a perishable product between producers and

processors. The perishable nature of milk has also led to the development of a unique futures contract that is cash settled to the class III price released by the USDA. Most futures contracts require physical delivery of the underlying commodity to an exchange approved warehouse. The complexities involved in making delivery mean that most producers offset their closing positions prior to expiration by assuming an equal and opposite position in the marketplace. Cash settlement however takes the hassle out of making delivery. Therefore, most hedgers of class III milk never offset their short class III futures positions. Cash settlement ensures zero basis risk because convergence between the cash and futures markets is ensured.

Producers that hedge class I milk do not have this luxury. Class I milk prices are based on a shorter survey period than class III prices. Furthermore class I prices are released on a different day than class III prices. This dissertation has outlined the relationship between advanced prices and the acceleration effect. The non storable nature of milk means that for class I hedgers offsetting their short position on the appropriate day is critical to minimizing the impact of the acceleration effect and maximizing hedge effectiveness. The problem that remains to be solved will arise only after producers attempt to use the hedging recommendations contained in this dissertation.

Florida producers could face significant liquidity risk when trying to exit the market. The average Florida dairy enrolled in DBAP in 2001, milked 1078 cows and averaged 45 pounds of milk. If this average Florida dairy attempted to hedge 80% of its expected milk production then the dairy producer would need to offset 7 contracts on the day class I prices are released. While it is not likely that one dairy would significantly move prices, if as few as 10 average sized Florida dairies attempted to offset their futures

position on the day that class I prices were released they could easily double the volume of trades for that day. This is problematic because each of the dairy producers would be trying to offset their short market positions with an equal opposite long position. These producers will be forced to compete with one another for short trades. The result is that if only a few Florida producers adopt the hedging recommendations presented in this dissertation, then they could conceivable increase milk prices on the day that class I prices are released. The increased milk prices would reduce the producer's hedge effectiveness yielding an expected loss from the producer's hedge position.

The Monte Carlo simulation model developed in this dissertation offers the flexibility necessary to tailor risk mitigating needs to the individual producer. Caution should be exercised to ensure that producers attempting to hedge will not lose money when they try to liquidate their market positions.

APPENDIX A
STOCHASTIC MILK UTILIZATION IN FLORIDA

Introduction

Prior to federal order reform in 2000, Florida was compromised of three marketing orders. Upper Florida (Order 6), Tampa Bay (Order 12), and Southeastern Florida (Order 13) each had their own orders. The orders were consolidated into one order because of overlapping sales, which resulted from a lack of natural boundaries between orders. Therefore, because of this structural change in the way milk was marketed by the industry, the regression used to estimate milk utilization is based on historical utilizations after order reform began in January 2000 and ending with data from December 2003.

The regression used to predict monthly milk utilization, Y_{ij}, where i equals an integer 1 to 4 corresponding with the class utilization and j equals an integer 1 to 48 corresponding with the month, where January 2000 equals one. The regression includes utilization from each of the four milk classes. Twenty explanatory variables are included in the model. For each class a trend variable and four intercept terms were included (one for each quarter). Quarterly variables were included to capture the expected seasonality in class utilization. A seasonal change in utilization is expected because of seasonal changes in milk production (Washington et al. 2000). Class I utilization is more inelastic than other class utilizations in the short run (Kilmer 1987). Therefore, during the summer months when production decreases, the amount of milk utilized in the class I market remains relatively stable compared to the other markets and the percent utilization is expected to increase. The average detrended utilization for each class by quarter is

captured by the intercept terms. In addition the trend variable is included to capture gradual changes over time in the utilization of milk in Florida (Putnam and Allshouse 2003).

Econometric Model to Predict Milk Utilization

The utilization estimate for a given monthly class utilization can be found by minimizing the sum of squared residuals in matrices

$$\begin{bmatrix} Y_{1,1} \\ Y_{1,2} \\ \vdots \\ Y_{1,47} \\ Y_{2,1} \\ Y_{2,2} \\ \vdots \\ Y_{2,47} \\ Y_{3,1} \\ Y_{3,2} \\ \vdots \\ Y_{3,47} \\ Y_{4,1} \\ Y_{4,2} \\ \vdots \\ Y_{4,47} \end{bmatrix} = \begin{bmatrix} 1 & 0/1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0/1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 47 & 0/1 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0/1 & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 2 & 0/1 & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & 47 & 0/1 & 0 & 0 & \vdots & \vdots \\ \vdots & \vdots & 0 & 0 & 1 & 0/1 & \vdots & \vdots \\ \vdots & \vdots & 0 & 0 & 2 & 0/1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 47 & 0/1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & 1 & 0/1 \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & 2 & 0/1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 47 & 0/1 \end{bmatrix} \begin{bmatrix} TRND^{I} \\ Q^{I}_{1-4} \\ TRND^{II} \\ Q^{II}_{1-4} \\ TRND^{III} \\ Q^{III}_{1-4} \\ TRND^{IV} \\ Q^{IV}_{1-4} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \vdots \\ \varepsilon_{1,47} \\ \varepsilon_{2,1} \\ \varepsilon_{2,2} \\ \vdots \\ \varepsilon_{2,47} \\ \varepsilon_{3,1} \\ \varepsilon_{3,2} \\ \vdots \\ \varepsilon_{3,47} \\ \varepsilon_{4,1} \\ \varepsilon_{4,2} \\ \vdots \\ \varepsilon_{4,47} \end{bmatrix}, \qquad \text{(A-1)}$$

which can be summarized as

$$\mathbf{Y} = \mathbf{XB} + \boldsymbol{\varepsilon} . \qquad \text{(A-2)}$$

The coefficient vector can be found by

$$Min \ (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}) \qquad \text{(A-3)}$$

subject to

$$\mathbf{R\beta} = \mathbf{q}\,, \tag{A-4}$$

where the matrix **R** equals

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}, \tag{A-5}$$

and restricts the sum of the trend terms for each of the four classes to 0 and the sum of the class intercept terms for each quarter to 1. These restrictions are identified in matrix **q**

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \tag{A-6}$$

Following Greene (2000), the restricted ordinary least squares estimator can be derived using the Lagrangian (Greene 2000)

$$L(\boldsymbol{\beta}, \boldsymbol{\lambda}) = (\mathbf{Y} - \mathbf{X\beta})'(\mathbf{Y} - \mathbf{X\beta}) + 2\boldsymbol{\lambda}'(\mathbf{R\beta} - \mathbf{q})\,. \tag{A-7}$$

The solutions $\boldsymbol{\beta}^*$ and $\boldsymbol{\lambda}^*$ satisfy the following first order conditions

$$\frac{\partial L}{\partial \boldsymbol{\beta}} = -2\mathbf{X}'(\mathbf{Y} - \mathbf{X\beta}) + 2\mathbf{R}'\boldsymbol{\lambda} = 0 \tag{A-8}$$

and

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = 2(\mathbf{R\beta} - \mathbf{q}) = 0\,. \tag{A-9}$$

Dividing through by 2 and rearranging yields the expression of partitioned matrices

$$\begin{bmatrix} \mathbf{X'X} & \mathbf{R'} \\ \mathbf{R} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} \mathbf{X'Y} \\ \mathbf{q} \end{bmatrix}, \tag{A-10}$$

which can be summarized as

$$\mathbf{Wd}^* = \mathbf{v}\,. \tag{A-11}$$

Pre-multiplying both sides by the inverse of **W** yields the restricted ordinary least squares estimates (Greene 2000)

$$\mathbf{d}^* = \mathbf{W}^{-1}\mathbf{v}\,. \tag{A-12}$$

The restricted ordinary least square estimates are reported in table A-1. The average milk utilization in the class I market from January to March is 90.47%. During this same period classes II, III, and IV are expected to utilize 6.16, 1.58 and 1.79% respectively. The estimates for the trend coefficient indicate that class I utilization is expected to decrease by a statistically significant 0.0748% every month (one sided t test is significant at the 99% confidence interval). Decreases over time in class I utilization were offset by increases of 0.0315, 0.0339, and 0.0094% in class II, III, and IV respectively.

Two tailed tests reveal that the restrictions on quarter I, II and III were significant at the 0.001 confidence interval while the restrictions on quarter IV and on the trend were not significant at even the 10% confidence interval.

These data support the hypothesis that there is significant seasonal variation in class utilization. The corresponding F statistic used to calculate the significance of seasonal variation equals

$$F(4,177) = \frac{\frac{0.08826 - 0.06197}{12}}{\frac{0.06197}{160}} = 5.657\,, \qquad \text{(A-13)}$$

where the degrees of freedom in the denominator equals the number of observations (192) minus the number of the explanatory variables (20) minus the number of restrictions (12) on the original model. The F statistic indicates that variation in the quarterly class intercepts is significant at the 0.001 confidence interval.

These data support the hypothesis that class I utilization increases during the summer months. From July through September the class I utilization is expected to increase by 4.91% over class I milk utilization during the second quarter. Class I milk utilization is numerically different during the second quarter than during the other quarters.

Table A-1. Restricted ordinary least squares milk class utilization parameter labels, variables and estimates

Label	Parameter	Estimate	Error	t Value	One Tail
Class I trend	$TRND^{I}$	-0.00077	0.000174	-4.42	<.0001
Class II trend	$TRND^{II}$	0.000317	0.000174	1.82	0.1406
Class III trend	$TRND^{III}$	0.000347	0.000174	1.99	0.0952
Class IV trend	$TRND^{IV}$	0.000104	0.000174	0.6	1.0992
Class I, Quarter I	Q_1^{I}	0.90516	0.00583	155.25	<.0001
Class I, Quarter II	Q_2^{I}	0.86737	0.00616	140.9	<.0001
Class I, Quarter III	Q_3^{I}	0.91651	0.00651	140.84	<.0001
Class I, Quarter IV	Q_4^{I}	0.91292	0.00688	132.68	<.0001
Class II, Quarter I	Q_1^{II}	0.06152	0.00583	10.55	<.0001
Class II, Quarter II	Q_2^{II}	0.06687	0.00616	10.86	<.0001
Class II, Quarter III	Q_3^{II}	0.06391	0.00651	9.82	<.0001
Class II, Quarter IV	Q_4^{II}	0.06042	0.00688	8.78	<.0001
Class III, Quarter I	Q_1^{III}	0.01567	0.00583	2.69	0.0158
Class III, Quarter II	Q_2^{III}	0.02858	0.00616	4.64	<.0001
Class III, Quarter III	Q_3^{III}	0.00493	0.00651	0.76	0.8986
Class III, Quarter IV	Q_4^{III}	0.00642	0.00688	0.93	0.7042
Class IV, Quarter I	Q_1^{IV}	0.01765	0.00583	3.03	0.0056
Class IV, Quarter II	Q_2^{IV}	0.03718	0.00616	6.04	<.0001
Class IV, Quarter III	Q_3^{IV}	0.01465	0.00651	2.25	0.0512
Class IV, Quarter IV	Q_4^{IV}	0.02024	0.00688	2.94	0.0074
Quarter I Constraint	RESTRICT	-1.96E-17	0.03241	0	1.0000*
Quarter II Constraint	RESTRICT	-4.43E-16	0.03241	0	1.0000*
Quarter III Constraint	RESTRICT	-1.21E-16	0.03241	0	1.0000*
Quarter IV Constraint	RESTRICT	-0.00913	0.03241	-0.28	0.7792*
Trend Constraint	RESTRICT	-0.42887	1.82427	-0.24	0.8149*

* Two tailed test using beta distribution

Table A-2. Milk utilization data (January 2000 through December 2003)

		2000	2001	2002	2003
Class I Milk Utilization	January	87.97%	89.91%	90.29%	88.91%
	February	90.53%	89.57%	90.08%	87.10%
	March	87.34%	91.30%	88.35%	86.40%
	April	81.75%	87.63%	87.74%	77.76%
	May	84.34%	90.40%	87.43%	79.13%
	June	88.74%	91.28%	86.19%	77.25%
	July	88.51%	87.41%	90.14%	86.67%
	August	91.91%	90.96%	91.98%	87.78%
	September	91.66%	89.39%	89.20%	90.23%
	October	89.79%	91.96%	91.74%	90.33%
	November	90.14%	90.17%	90.42%	86.65%
	December	86.45%	88.81%	84.62%	86.78%
Class II Milk Utilization	January	5.95%	6.42%	5.95%	6.72%
	February	6.18%	7.20%	6.51%	7.30%
	March	7.30%	6.73%	7.04%	8.13%
	April	7.37%	6.93%	7.65%	7.03%
	May	7.85%	7.01%	8.61%	5.67%
	June	7.97%	6.98%	8.62%	7.30%
	July	7.59%	7.46%	7.99%	8.14%
	August	5.67%	7.36%	6.51%	9.19%
	September	6.00%	5.83%	6.75%	8.09%
	October	5.68%	6.04%	6.82%	7.65%
	November	6.08%	5.99%	7.84%	9.08%
	December	7.04%	5.13%	7.64%	7.63%
Class III Milk Utilization	January	3.02%	2.15%	2.12%	2.36%
	February	2.14%	1.87%	0.64%	3.16%
	March	1.90%	1.41%	2.89%	3.48%
	April	2.38%	2.69%	2.98%	8.23%
	May	2.34%	2.18%	1.49%	7.51%
	June	1.94%	1.58%	2.75%	7.81%
	July	2.35%	0.70%	1.74%	3.51%
	August	1.72%	0.55%	1.17%	0.45%
	September	1.69%	0.54%	2.03%	0.30%
	October	2.61%	0.35%	0.18%	0.14%
	November	1.83%	2.06%	1.07%	0.31%
	December	2.58%	0.52%	4.58%	2.64%

Table A-2 . -Continued

		2000	2001	2002	2003
Class IV Milk Utilization	January	3.06%	1.52%	1.64%	2.01%
	February	1.15%	1.36%	2.77%	2.44%
	March	3.46%	0.56%	1.72%	1.99%
	April	8.50%	2.75%	1.63%	6.98%
	May	5.47%	0.41%	2.47%	7.69%
	June	1.35%	0.16%	2.44%	7.64%
	July	1.55%	4.43%	0.13%	1.68%
	August	0.70%	1.13%	0.34%	2.58%
	September	0.65%	4.24%	2.02%	1.38%
	October	1.92%	1.65%	1.26%	1.88%
	November	1.95%	1.78%	0.67%	0.31%
	December	3.93%	5.54%	3.16%	2.95%

APPENDIX B
PRODUCTION RISK

Florida Milk Production

Nearly 2000 years ago tribes settled in region along the Rhine delta that is today known as the Netherlands. Settlement in the new land required the development of animals that were able to survive and flourish in the new environment. One of those animals was the Holstein-Friesian dairy cow. The unequaled production and efficient feed milk conversion made the Holstein dairy cow a natural choice for settlers in the new world. Today, the Holstein is by far the dominant source of milk in the United States. European cattle (*bos taurus*) are differentiated from cattle that evolved in Asia (*bos indicus*) in two regards. European cattle, such as Holsteins, lack the sweat glands and loose skin of their Asian cousins. These two attributes help Asian cattle to be more tolerant of hot, humid climates. Despite the sensitivity to the hot, humid Florida summers, the Holstein cow is still the favored milk-producing breed in Florida because of its highly efficient milk to feed conversion ratio. Sensitivity to hot, humid Florida summers leads to significant seasonality in Florida milk production. Milk production per cow is typically the highest during February, March, and April and lowest during August, September, and October (Washington et al. 2000). Figure B-1 illustrates the seasonality in milk production per cow in Florida over time using monthly Florida Agricultural Statistics Service (FASS) data.

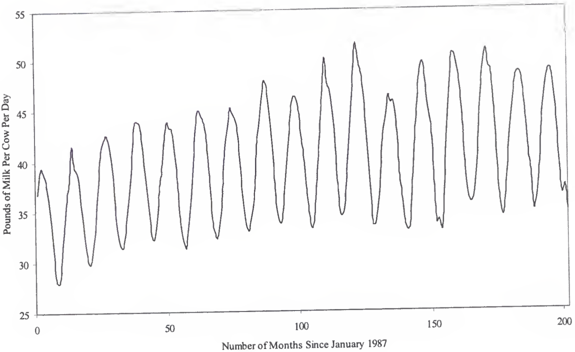

Figure B-1. Seasonal Florida milk production per cow (January 1987-December 2003)[1]

Econometric Model Used to Predict Milk Production Per Cow

Production risk requires an estimate of the probabilities of deviations from expected future production. In order to accomplish this monthly production data from January 1987 to December 2003 were used. It is assumed that 17 years of production data will adequately define production risk for the purposes of this model.

Efficiency gains from advances in nutrition and genetic gains from artificial insemination have helped to continually improve the genetic potential of Holsteins in the United States. A result of this is that milk production per cow increases predictably over time. The gradual increase over time was modeled using the natural log of time. Seasonality was accounted for by using a cosine and sine estimator. The modeling of

[1] Milk production data taken from http://www.nass.usda.gov/fl/lvstk/ldpsum02/02ldp06.htm and http://www.nass.usda.gov/fl/lvstk/ldpsum96/96ldp6.htm

seasonality employed here closely parallels the approach used by Washington et al. (2000).

Milk production per cow per day can be estimated for a given month by

$$PPC_t = INT + PCL * Ln(EM) + SEST * \sin\left[\left(\frac{f * EM}{n}\right)2\pi\right] + CEST * \cos\left[\left(\frac{f * EM}{n}\right)2\pi\right] + e_t, \qquad \text{(B-1)}$$

where *INT* equals the intercept term, *EM* equals the number of elapsed months sinse January 1987, *PCL* equals the regression coefficient of the natural log of *EM*, *SEST* equals the sine estimator, *n* equals the number of observations, *f* equals the frequency or *n*/12, and *CEST* equals the cosine coefficient.

Analysis of variance reveals that 94.07% of the variation in milk production per cow per day can be accounted for by these variables. Parameter estimates can be found in table B-1. All of the parameter estimates were found to be significant at the 0.001 confidence interval.

Table B-1. Restricted ordinary least square parameter estimates and their statistical significance

Label	Variable	Estimate	Error	t Value	Pr>|t|
Intercept	*INT*	28.6977535	0.443167614	64.7559	3.841E-136
Natural Log	*PCL*	2.61545329	0.09984411	26.1953	1.43137E-66
Sine Estimator	*SEST*	6.92192602	0.134526501	51.4539	2.6482E-117
Cosine Estimator	*CEST*	0.55475290	0.152322427	3.64196	0.00034464

Forecasting Milk Production and Production Risk

The first step to estimating production risk for Florida's dairy producers is to detrend the data using the natural log of *EM*. This was accomplished by subtracting the predicted value (assuming *SEST* and *CEST* equal zero) from the actual value. Subtracting

the trend removes variation that is due to the adoption of supply enhancing technologies. The variation that remains in production per cow is due to seasonal changes. The detrended data is presented in figure B-2.

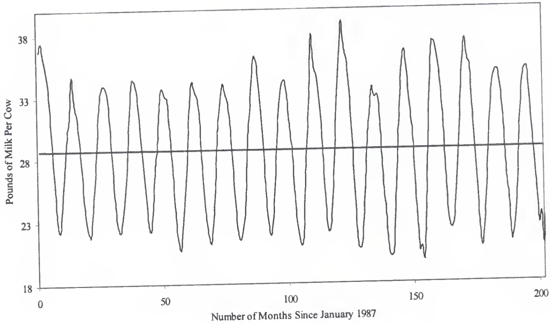

Figure B-2. Detrended milk production per cow (January 1987-December 2003)

The sine estimates and cosine parameter estimates generated were relatively good. However, despite the accuracy of the sine and cosine functions, the historical average will be used as the basis for estimating production risk for this study because of its computational ease. The historical detrended data, sine estimator and historical average estimator are grouped by month in figure B-3.

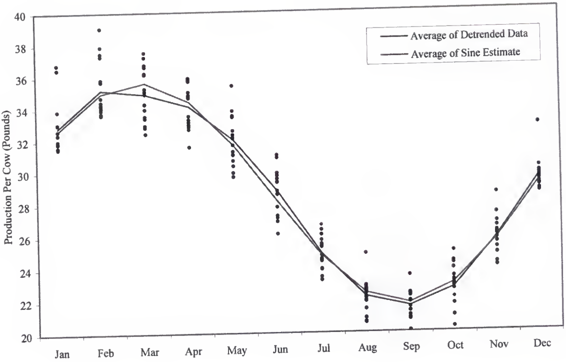

Figure B-3. Sine and moving average estimate of detrended milk production per cow.

APPENDIX C
MILK PRODUCTION EXPENSES FOR FLORIDA DAIRY PRODUCERS

The economic literature has shown that production risk impacts the producer's optimal revenue hedge ratio (Lapan and Moschini 1994). Hedging net income is slightly more complicated because decreases in production are not accompanied by proportional decreases in expenses because of fixed costs and economies of scale. The primary objective of constructing econometric estimates of total costs per hundredweight was to determine the sensitivity of total cost per hundredweight to changes in milk production per cow. The parameters yielded from the regression presented in this appendix were used in the hedging Monte Carlo simulation model presented in chapter 4.

Dairy Business Analysis Project Data

The data used to estimate producer expenses was collected by the Dairy Business Analysis Project (DBAP) at the University of Florida. Data reported by DBAP conforms to the recommendations of the Farm Financial Standards Council (FFSC). Accrual adjustments were made to cash receipts and expenditures to account for changes in inventory, prepaid expenses, depreciation, accounts receivable and accounts payable. The depreciation of property and equipment was estimated using data from tax returns. Assets were valued using conservative estimates of current market values.

In order to ensure the integrity of the data used, a balancing constraint was imposed on the data used to calculate expense estimates. The balancing constraint is based on

dairies where at least 90% of the changes in cash flows and equity could be accounted for by either the statement of cash flows, or the statement of owner's equity. The cash flow statement reconciles changes in cash from January 1 to December 31 due to the financing, investing, and operating functions of the dairy. The equity statement reconciles changes to owner's equity through reported changes to retained capital and valuation of calculated equity (de Vries et al. 2001).

Some dairies joined DBAP during latter years, while other dairies went out of business after the first few years on DBAP. Therefore, the data set was unbalanced. Only dairies with at least two years of data were used in the calculations. Furthermore, the dairies that were included were asked to join DBAP and therefore the data does not represent a random sample of Florida dairies.

Econometric Estimates of Expense Variation

El Osta and Johnson found that cow productivity, purchased feed costs, and region were among the most significant determinants explaining the variation in net returns per hundredweight (El Osta and Johnson 1998). Since net returns per hundredweight are, for the most part, determined by the cost of production, the developed econometric model was specified using similar variables. Total cost per hundredweight is the measure of cost of production used as the dependent variable in the model. Labor costs per cow were also included as an additional explanatory variable because of the relatively important significance illustrated in figure C-1. Econometric total cost estimates were specified by region. No data were available for South Florida dairies. Either data did not exist or no data met the selection criteria from Martin, Palm Beach, Broward, Dade, Monroe, Charlotte, Glades, Lee, Hendry, or Collier counties. Florida counties North of

Citrus, Marion, and Volusia were considered North Florida while the remaining counties were classified as Central Florida.

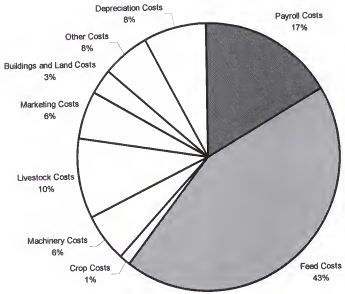

Figure C-1. Relative cost of milk production for Florida dairy producers in 2001

The cross-sectional time-series model used to estimate total cost per cwt for a given firm *j* at year *Y*, $\left(\frac{TC}{CWT}\right)_{j,Y}$, was constructed as follows:

$$\left(\frac{TC}{CWT}\right)_{j,Y} = f\begin{pmatrix} ND_j, CD_j, North, PPC_{j,Y} - \overline{PPC}, (PPC_{j,Y} - \overline{PPC})^2, \\ \left(\frac{FEED_{j,Y}}{COW_{j,Y}} - \frac{\overline{FEED}}{\overline{COW}}\right), \left(\frac{LABOR_{j,Y}}{COW_{j,tY}} - \frac{\overline{LABOR}}{\overline{COW}}\right), \\ \left(COW_{j,Y} - \overline{COW}\right) \end{pmatrix}, \quad \text{(C-2)}$$

where *North* is a binary variable identifying either the northern (1) or central region (0), ND_j and CD_j are binary variables used to model the fixed effects for each northern and central dairy respectively, $PPC_{j,Y}$ is the production per cow for firm *j* at year *Y* measured

in pounds, $\overline{PPC}$ is the average production per cow, $FEED_{j,Y}$ is the feed cost per cow for firm *j* at year *Y*, $COW_{j,Y}$ is the number of cows milked by firm *j* during year *Y*, $\dfrac{\overline{FEED}}{\overline{COW}}$ is the average feed cost per cow for Florida producers, $\dfrac{LABOR_{j,Y}}{COW_{j,Y}}$ is the labor cost per cow for firm *j* at year Y, $\dfrac{\overline{LABOR}}{\overline{COW}}$ is the average labor cost per cow.

The fixed effect approach or the random effects approach can be used to estimate (C-2) (Greene 2000) The approach used here parallels most closely the fixed approach . The DBAP data consists of unbalanced cross-sectional and time series data. Equation (C-2) can be estimated as

$$\begin{aligned}\left(\frac{TC}{CWT}\right)_{j,Y} &= \beta_0 + \sum_{j=1}^{16} \beta_j ND_j + \sum_{j=17}^{36} \beta_j NC_j + \beta_{37} North + \\ &\beta_{38}\left(PPC_{j,Y} - \overline{PPC}\right) + \beta_{39}\left(PPC_{j,Y} - \overline{PPC}\right)^2 + \beta_{40}\left(\frac{FEED_{j,Y}}{COW_{j,Y}} - \frac{\overline{FEED}}{\overline{COW}}\right) +, \qquad \text{(C-3)} \\ &\beta_{41}\left(\frac{LABOR_{j,Y}}{COW_{j,Y}} - \frac{\overline{LABOR}}{\overline{COW}}\right) + \beta_{42}\left(COW_{j,Y} - \overline{COW}\right) + \varepsilon_{j,Y}\end{aligned}$$

where *j* represents the *j*th dairy in the sample in year *Y*=1995,1996…2004. There are 17 dairies in the northern region and 20 dairies in the central region. The means are subtracted from each of the explanatory variables in an attempt to make β_0 more meaningful. When each of the variables is corrected by its respective mean then β_0 takes a value equal to the mean total cost for a central Florida DBAP dairy. The variable *NORTH* is a dummy variable that equals one when the dairy is from North Florida. The mean for North Florida DBAP dairies equals $\beta_0 + \beta_{36}$.

Typically a dummy variable would be used for each firm. When a dummy variable is used for each firm the last firm is arbitrarily dropped, in order to avoid the dummy variable trap (singular matrix). When the last variable is dropped it is effectively constrained to equal zero. Another approach might be to set the sum of β_1 to β_{16} and the sum of β_{17} to β_{35} to zero. The variable ND_j=D_j-D_{17} for j=1,...16 and variable NC_j=D_j-D_{37} for j=17,...36 where the D_j equals a dummy variable for each of the 37 dairies. The variables D_{17} and D_{37} are dummy variables for the last Northern dairy in the data set and last central dairy in the data set respectively. The variable ND_j captures the firm specific fixed effects for North Florida dairies and the variable NC_j captures the firm specific fixed effects for central Florida dairies. The advantage of using this approach versus the traditional approach is that β_0 retains the significance of being the mean total cost per hundredweight (Briz and Ward 1998) .

The parameter estimates of equation (C-3) are contained in table C-1. The findings suggest that 76.6% of the variation in total cost per hundredweight is explained by the explanatory variables. The model intercept equals 16.54. The intercept equals the average cost for Florida producers to produce a hundredweight of milk from 1995 to 2001. The parameter estimates for the fixed effects of North and Central Florida dairies show significant variation. The fixed effects will not be discussed further because they are not utilized to build the Monte Carlo simulation model in chapter 4. The regional variable β_{37} was not found to be statistically significant. The parameter estimate of β_{38} equals –0.18 and was significant at the 0.01% confidence interval. The negative sign on the estimate indicates that as production per cow increases above the mean 46.93[1] pound

[1] The average production per cow was estimated using DBAP data.

per cow total costs drop. The decrease in costs as production per cow increases suggests that margins will increase by $0.18 with each pound increase in average milk production per cow. This is equivalent to stating that significant economies of scale exist in the Florida dairy industry. The positive sign on the quadratic term β_{39} indicate that gains due to increasing production per cow gradually taper off suggesting that gains from economies of scale exist diminish with each additional pound above the mean 46.93. The diminishing returns to economies of scale are significant at the 0.05 confidence interval. The cost parameters β_{40} and β_{41} were both significant at the 0.001 confidence interval . This indicates that total costs per hundredweight are expected to increase as costs extend beyond the mean feed cost per cow per year of $1377.40 and the mean labor cost per cow per year$457.03[2] per cow per year respectively. Finally the deviation in the number of cows from the mean DBAP herd (1230) was not shown to be statistically significant.

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[2] The average feed and labor costs per cow were estimated using DBAP data.

Table C-1 Changes in milk production costs due to changing production

Label	Par	Estimate	Error	t Value	Pr t>|t|
Intercept	β_0	16.5444	0.36534	45.28	<0.0001
Fixed Effect Northern (Dairy # 1)	β_1	-0.7344	2.30161	-0.32	0.7504
Fixed Effect Northern (Dairy # 2)	β_2	-0.0974	0.99276	-0.1	0.9221
Fixed Effect Northern (Dairy # 3)	β_3	-1.3643	2.3021	-0.59	0.5549
Fixed Effect Northern (Dairy # 4)	β_4	-0.1976	0.72357	-0.27	0.7853
Fixed Effect Northern (Dairy # 5)	β_5	-2.4511	0.8123	-3.02	0.0033
Fixed Effect Northern (Dairy # 6)	β_6	-2.1428	0.80633	-2.66	0.0093
Fixed Effect Northern (Dairy # 7)	β_7	-0.4638	1.0479	-0.44	0.6591
Fixed Effect Northern (Dairy # 8)	β_8	3.9031	0.82008	4.76	<0.0001
Fixed Effect Northern (Dairy # 9)	β_9	1.6267	0.77039	2.11	0.0375
Fixed Effect Northern (Dairy # 10)	β_{10}	-2.3102	0.86511	-2.67	0.009
Fixed Effect Northern (Dairy # 11)	β_{11}	-0.2603	1.02645	-0.25	0.8004
Fixed Effect Northern (Dairy # 12)	β_{12}	-0.7047	0.87659	-0.8	0.4236
Fixed Effect Northern (Dairy # 13)	β_{13}	3.6553	0.92938	3.93	0.0002
Fixed Effect Northern (Dairy # 14)	β_{14}	0.9532	0.53931	1.77	0.0806
Fixed Effect Northern (Dairy # 15)	β_{15}	-0.0291	0.91771	-0.03	0.9748
Fixed Effect Northern (Dairy # 16)	β_{16}	0.2955	0.95751	0.31	0.7583
Fixed Effect Central (Dairy # 17)	β_{17}	1.7458	1.00522	1.74	0.0859
Fixed Effect Central (Dairy # 18)	β_{18}	-1.5961	0.88907	-1.8	0.076
Fixed Effect Central (Dairy # 19)	β_{19}	-3.2189	1.04243	-3.09	0.0027
Fixed Effect Central (Dairy # 20)	β_{20}	0.15837	1.337	0.12	0.906
Fixed Effect Central (Dairy # 21)	β_{21}	-3.0890	0.69663	-4.43	<0.0001
Fixed Effect Central (Dairy # 22)	β_{22}	-1.8382	0.66952	-2.75	0.0073
Fixed Effect Central (Dairy # 23)	β_{23}	0.3079	0.97379	0.32	0.7526
Fixed Effect Central (Dairy # 24)	β_{24}	0.2091	1.41186	0.15	0.8826
Fixed Effect Central (Dairy # 25)	β_{25}	0.6103	1.70325	0.36	0.721
Fixed Effect Central (Dairy # 26)	β_{26}	-0.1499	0.81737	-0.18	0.8548
Fixed Effect Central (Dairy # 27)	β_{27}	2.1656	7.96115	0.27	0.7862
Fixed Effect Central (Dairy # 28)	β_{28}	-0.0855	1.95599	-0.04	0.9652
Fixed Effect Central (Dairy # 29)	β_{29}	0.4112	0.73586	0.56	0.5777
Fixed Effect Central (Dairy # 30)	β_{30}	0.1617	0.45691	0.35	0.7242
Fixed Effect Central (Dairy # 31)	β_{31}	0.9025	0.64445	1.4	0.1649
Fixed Effect Central (Dairy # 32)	β_{32}	-2.0942	1.08989	-1.92	0.0579
Fixed Effect Central (Dairy # 33)	β_{33}	0.9055	0.84653	1.07	0.2876
Fixed Effect Central (Dairy # 34)	β_{34}	0.1454	1.48931	0.1	0.9224
Fixed Effect Central (Dairy # 35)	β_{35}	0.9097	0.85012	1.07	0.2874
Fixed Effect Central (Dairy # 36)	β_{36}	1.1127	0.69872	1.59	0.1148
Dummy Variable for North	β_{37}	0.7767	0.57973	1.34	0.1837
Deviation from mean PPC	β_{38}	-0.1843	0.03182	-5.79	<0.0001
Squared Deviation from mean PPC	β_{39}	0.0029	0.00136	2.16	0.0336
Deviation from mean $feed/cow	β_{40}	0.0053	64931	8.31	<0.0001
Deviation from mean $labor/cow	β_{41}	0.0060	0.00154	3.91	0.0002
Deviation from mean Cow	β_{42}	-0.00008	0.000821	-0.09	0.9252

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BIOGRAPHICAL SKETCH

Michael Zylstra grew up on a dairy farm in Modesto, which is located in California's great Central Valley. He attended Ripon Christian High School. Upon graduation from high school, and some encouragement from his parents, he decided to attend the California Polytechnic State University as a Dairy Science major in 1994. Nestled in the foothills of California's Central Coast, San Luis Obispo is the perfect place to study. After graduating with his bachelors in 1997, he continued at Cal Poly in the college of business earning a MBA in 1999. The second year of his MBA was financed by teaching multiple sections of an undergraduate computer applications course. He enjoyed teaching in the university environment. During his Ph.D. studies in the Food and Resource Economics, at the University of Florida, he enjoyed his interaction with fellow graduate students.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Richard L. Kilmer, Chair
Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Albert de Vries
Assistant Professor of Animal Science

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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